AP CALCULUS BC • EXERCISE SHEET

1.15 Connecting Limits at Infinity and Horizontal Asymptotes

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **connecting limits at infinity and horizontal asymptotes**.

You must be able to:

- relate a **limit at infinity** 无穷远处极限 to a **horizontal asymptote** 水平渐近线
- find it from the ratio of leading terms

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits at infinity

$\lim_{x\to\infty} f(x) = L$ means $y = L$ is a horizontal asymptote. For a rational function, compare leading terms:

- degree(top) < degree(bottom) $\to y = 0$;
- degrees equal $\to y = \dfrac{\text{leading coeff of top}}{\text{leading coeff of bottom}}$;
- degree(top) > degree(bottom) $\to$ no horizontal asymptote.

■ Example

$\lim_{x\to\infty} \dfrac{1}{x} = 0$, so $y = 0$ is a horizontal asymptote.

2 Practice

2.1 State what $\lim_{x\to\infty} f(x) = L$ indicates. [1]

2.2 For $f(x) = \dfrac{1}{x}$, state $\lim_{x\to\infty} f(x)$. [1]

2.3 For $f(x) = \dfrac{2x}{x+1}$, state the horizontal asymptote. [2]

3 Exam-style questions

3.1 $\lim_{x\to\infty} f(x) = L$ means $y = L$ is a [1]

- **A** vertical asymptote
- **B** horizontal asymptote
- **C** hole
- **D** root

3.2 $\lim_{x\to\infty} \dfrac{1}{x}$ equals [1]

- **A** 1
- **B** 0
- **C** ∞
- **D** -1

3.3 $f(x) = \dfrac{3x^2}{x^2+1}$.

(a) Compare the degrees. [1]

(b) Take the ratio of the leading coefficients. [1]

(c) State the horizontal asymptote. [1]

4 Go further

- work through the **1.15 Connecting Limits at Infinity and Horizontal Asymptotes** lesson on the **Learn** page;

- read the **Limits and Continuity** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 the graph has a horizontal asymptote $y = L$.

2.2 0.

2.3 $y = 2$ (equal degrees, ratio 2/1).

3.1 B.

3.2 B.

3.3 (a) equal (both degree 2). (b) $3/1 = 3$. (c) $y = 3$.