

1.14 Connecting Infinite Limits and Vertical Asymptotes

Name: _____ Class: _____ Date: _____ Total: 9 marks

KEY WORDS vertical asymptote 垂直渐近线 • infinite limit 无穷极限 • limit 极限
• horizontal asymptote 水平渐近线 • rational function 有理函数

Objective

Build the skills to answer exam questions on **connecting infinite limits and vertical asymptotes**.

You must be able to:

- relate an **infinite limit** 无穷极限 at a to a **vertical asymptote** 垂直渐近线 $x = a$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Infinite limits

If $f(x) \rightarrow \pm\infty$ as $x \rightarrow a$, the line $x = a$ is a **vertical asymptote**. This happens where the denominator $\rightarrow 0$ but the numerator does not.

■ Example

$f(x) = \frac{1}{x-2}$ has a vertical asymptote at $x = 2$; as $x \rightarrow 2^+$, $f(x) \rightarrow +\infty$.

2 Practice

2.1 State what an infinite limit at a indicates. [1]

2.2 For $f(x) = \frac{1}{x}$, state the vertical asymptote. [1]

2.3 State where a rational function has a vertical asymptote. [2]

3 Exam-style questions

3.1 If $\lim_{x \rightarrow a} f(x) = \infty$, then $x = a$ is a [1]

- A** horizontal asymptote
- B** vertical asymptote
- C** hole
- D** root

3.2 $f(x) = \frac{1}{x-2}$ has a vertical asymptote at [1]

- A** $x = 0$
- B** $x = 2$
- C** $y = 2$
- D** $x = -2$

3.3 $f(x) = \frac{1}{x-5}$.

(a) Where is the denominator 0? [1]

(b) State the vertical asymptote. [1]

(c) State the behaviour of $\lim_{x \rightarrow 5^+} f(x)$. [1]

Solutions

2.1 the graph has a vertical asymptote there.

2.2 $x = 0$.

2.3 where the denominator is 0 but the numerator is not.

3.1 B.

3.2 B.

3.3 (a) $x = 5$. (b) $x = 5$. (c) $+\infty$.