AP CALCULUS BC • EXERCISE SHEET

1.14 Connecting Infinite Limits and Vertical Asymptotes

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **connecting infinite limits and vertical asymptotes**.

You must be able to:

- relate an **infinite limit** 无穷极限 at a to a **vertical asymptote** 垂直渐近线 $x = a$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Infinite limits

If $f(x) \to \pm\infty$ as $x \to a$, the line $x = a$ is a **vertical asymptote**. This happens where the denominator $\to 0$ but the numerator does not.

Example

$f(x) = \dfrac{1}{x-2}$ has a vertical asymptote at $x = 2$; as $x \to 2^+$, $f(x) \to +\infty$.

2 Practice

2.1 State what an infinite limit at a indicates. [1]

2.2 For $f(x) = \dfrac{1}{x}$, state the vertical asymptote. [1]

2.3 State where a rational function has a vertical asymptote. [2]

3 Exam-style questions

3.1 If $\lim_{x \to a} f(x) = \infty$, then $x = a$ is a [1]

- **A** horizontal asymptote
- **B** vertical asymptote
- **C** hole
- **D** root

3.2 $f(x) = \dfrac{1}{x - 2}$ has a vertical asymptote at [1]

- **A** $x = 0$
- **B** $x = 2$
- **C** $y = 2$
- **D** $x = -2$

3.3 $f(x) = \dfrac{1}{x - 5}$.

(a) Where is the denominator 0? [1]

(b) State the vertical asymptote. [1]

(c) State the behaviour of $\lim_{x \to 5^+} f(x)$. [1]

4 Go further

- work through the **1.14 Connecting Infinite Limits and Vertical Asymptotes** lesson on the **Learn** page;
- read the **Limits and Continuity** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 the graph has a vertical asymptote there.

2.2 $x = 0$.

2.3 where the denominator is 0 but the numerator is not.

3.1 B.

3.2 B.

3.3 (a) $x = 5$. (b) $x = 5$. (c) $+\infty$.