

1.11 Defining Continuity at a Point

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS continuity 连续性 • continuous 连续 • limit 极限 • discontinuous 间断

Objective

Build the skills to answer exam questions on **defining continuity at a point**.

You must be able to:

- state the three conditions for **continuity** 连续性 at a point
- test continuity using $\lim_{x \rightarrow a} f(x) = f(a)$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Continuity at a point

f is continuous at a when all three hold:

1. $f(a)$ is defined;
2. $\lim_{x \rightarrow a} f(x)$ exists;
3. they are equal: $\lim_{x \rightarrow a} f(x) = f(a)$.

■ Example

If $\lim_{x \rightarrow 2} f(x) = 5$ and $f(2) = 5$, then f is continuous at 2.

2 Practice

2.1 State the three conditions for continuity at a . [1]

2.2 State the single equation summarizing continuity at a . [1]

2.3 If $\lim_{x \rightarrow 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

3 Exam-style questions

3.1 f is continuous at a if and only if [1]

- **A** $f(a)$ is defined
- **B** $\lim_{x \rightarrow a} f(x) = f(a)$
- **C** the limit exists only
- **D** f is linear

3.2 If $\lim_{x \rightarrow a} f(x)$ exists but $\neq f(a)$, then f is [1]

- **A** continuous at a
- **B** discontinuous at a
- **C** linear
- **D** undefined everywhere

3.3 $f(2) = 5$ and $\lim_{x \rightarrow 2} f(x) = 5$.

(a) Is $f(2)$ defined? [1]

(b) Does the limit exist? [1]

(c) Is f continuous at 2? [1]

Solutions

2.1 $f(a)$ is defined; the limit exists; the limit equals $f(a)$.

2.2 $\lim_{x \rightarrow a} f(x) = f(a)$.

2.3 yes — the limit exists, $f(2)$ is defined, and they are equal.

3.1 B.

3.2 B.

3.3 (a) yes. (b) yes. (c) yes.