Past-paper walk-through

Finding the Area of the Region Bounded by Two Polar Curves ▪ 9.9

eXercise

Questions in this set

- 2025 Q2
- 2019 Q2
- 2018 Q5
- 2017 Q2
- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

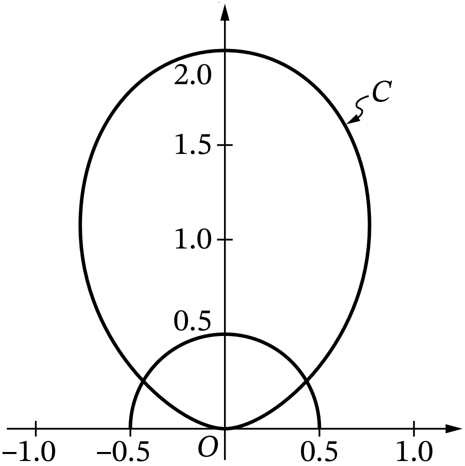
2025 ▪ Q2

Curve C is defined by the polar equation $r(\theta) = 2\sin^2\theta$ for $0 \le \theta \le \pi$. Curve C and the semicircle $r = \dfrac{1}{2}$ for $0 \le \theta \le \pi$ are shown in the xy-plane.
[Graph in the xy-plane, x-axis from -1.0 to 1.0, y-axis from 0 to 2.0 (gridlines at $0.5, 1.0, 1.5, 2.0$). Curve C (labeled with a squiggle and "C") is a large loop-shaped curve passing through the origin O, extending up to about $y = 3$ in shape but shown/gridded to about $y = 2.0$ at its widest visible extent, symmetric about the y-axis, touching the origin and bulging outward to about $x = \pm 1$ near the bottom, reaching its top around $(0, 2)$-ish region. A smaller semicircle of radius $\frac{1}{2}$ (equation $r = \frac{1}{2}$) is nested inside curve C near the origin, spanning from about $(-0.5, 0)$ to $(0.5, 0)$ and bulging up to about $y = 0.5$, overlapping the lower portion of curve C.]
(Note: Your calculator should be in radian mode.)

Question 2

(a) Find the rate of change of *r* with respect to θ at the point on curve *C* where $\theta = 1.3$. Show the setup for your calculations.

Question 2



Question 2

2025 ▪ Q2

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(Note: Your calculator should be in radian mode.)

Question 2

(b) Find the area of the region that lies inside curve C but outside the graph of the polar equation $r = \dfrac{1}{2}$. Show the setup for your calculations.

Question 2

2025 ▪ Q2

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(Note: Your calculator should be in radian mode.)

Question 2

(c) It can be shown that $\dfrac{dx}{d\theta} = 4\sin\theta\cos^2\theta - 2\sin^3\theta$ for curve C. For $0 \leq \theta \leq \dfrac{\pi}{2}$, find the value of θ that corresponds to the point on curve C that is farthest from the y-axis. Justify your answer.

Question 2

2025 ▪ Q2

Curve C is defined by the polar equation $r(\theta) = 2\sin^2\theta$ for $0 \leq \theta \leq \pi$. Curve C and the semicircle $r = \dfrac{1}{2}$ for $0 \leq \theta \leq \pi$ are shown in the xy-plane.

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(Note: Your calculator should be in radian mode.)

Question 2

(d) A particle travels along curve C so that $\dfrac{d\theta}{dt} = 15$ for all times t. Find the rate at which the particle's distance from the origin changes with respect to time when the particle is at the point where $\theta = 1.3$. Show the setup for your calculations.

Solution 2

(a)

Mark scheme

- $r(\theta) = 2\sin^2\theta \Rightarrow \dfrac{dr}{d\theta} = 4\sin\theta\cos\theta$. At $\theta = 1.3$:
  $\left.\dfrac{dr}{d\theta}\right|_{\theta=1.3} = 4\sin(1.3)\cos(1.3) = 1.031003 \approx 1.031$. (1 pt: answer with setup/differentiation shown, accurate to 3 decimals.)

Solution 2

(b)

Mark scheme

- Curve C meets $r = \frac{1}{2}$ at $\theta_1 = \frac{\pi}{6} = 0.523599$ and $\theta_2 = \frac{5\pi}{6} = 2.617994$. Area $= \dfrac{1}{2}\int_{\theta_1}^{\theta_2}\left[(r(\theta))^2 - \left(\frac{1}{2}\right)^2\right] d\theta = 2.066769 \approx 2.067$ (or 2.066). (1 pt: integrand including $(r(\theta))^2$; 1 pt: fully correct integrand, e.g. $(r(\theta))^2 - (1/2)^2$; 1 pt: correct final answer including limits and the $\frac{1}{2}$ factor.)

Solution 2

(c)

Mark scheme

- Set $\dfrac{dx}{d\theta} = 4\sin\theta\cos^2\theta - 2\sin^3\theta = 0 \Rightarrow \theta = 0.955317$ (equivalently $\arccos(1/\sqrt{3})$, $\arcsin(\sqrt{2/3})$, or $\arctan(\sqrt{2})$). Candidates test on $[0, \pi/2]$: $x(0) = 0$, $x(0.955317) = 0.769800$, $x(\pi/2) = 0$. The point farthest from the y-axis occurs at $\theta = 0.955$. (1 pt: considers $dx/d\theta = 0$; 1 pt: justification via global/candidates-test argument; 1 pt: correct answer with supporting work.)

Solution 2

(d)

Mark scheme

- $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$. At $\theta = 1.3$: $\left.\frac{dr}{dt}\right|_{\theta=1.3} = 1.031003 \cdot 15 = 15.465041 \approx 15.465$.
  (1 pt: sets up $dr/d\theta \cdot d\theta/dt$ symbolically or numerically; 1 pt: correct answer.)

Question 2

2019 ▪ Q2

Let S be the region bounded by the graph of the polar curve $r(\theta) = 3\sqrt{\theta}\sin\left(\theta^2\right)$ for $0 \leq \theta \leq \sqrt{\pi}$, as shown in the figure above.
[Figure: A polar-coordinate graph with axes x (horizontal) and y (vertical), origin labeled O. The shaded region S is a teardrop/loop shape starting at the origin, bulging out to the right and extending upward, bounded by the curve $r(\theta) = 3\sqrt{\theta}\sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$.]

(a) Find the area of S.

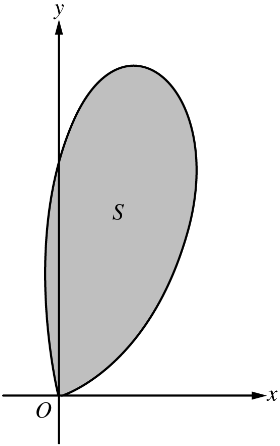
(b) What is the average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta}\sin\left(\theta^2\right)$ for $0 \leq \theta \leq \sqrt{\pi}$?

(c) There is a line through the origin with positive slope m that divides the region S into two regions with equal areas. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of m.

Question 2

(d) For $k > 0$, let $A(k)$ be the area of the portion of region S that is also inside the circle $r = k\cos\theta$. Find $\lim_{k\to\infty} A(k)$.

Question 2



Solution 2

(a)

Mark scheme

- Area of $S = \dfrac{1}{2}\int_0^{\sqrt{\pi}} (r(\theta))^2 \, d\theta = 3.534292$. The area of S is 3.534 (accept 3.535). Points: 1 for the integral, 1 for the answer.

Solution 2

(b)

Mark scheme

- Average distance $= \dfrac{1}{\sqrt{\pi} - 0} \int_0^{\sqrt{\pi}} r(\theta)\, d\theta = 1.579933$. The average distance from the origin to a point on the polar curve $r(\theta) = 3\sqrt{\theta}\sin(\theta^2)$ for $0 \leq \theta \leq \sqrt{\pi}$ is 1.580 (or 1.579). Points: 1 for the integral, 1 for the answer.

Solution 2

(c)

Mark scheme

- A line through the origin with slope m satisfies $\tan\theta = y/x = m \Rightarrow \theta = \tan^{-1} m$. This line divides S into two equal-area regions when $\frac{1}{2}\int_0^{\tan^{-1} m} (r(\theta))^2\,d\theta = \frac{1}{2}\left(\frac{1}{2}\int_0^{\sqrt{\pi}} (r(\theta))^2\,d\theta\right)$ (the area swept from $\theta = 0$ to $\theta = \tan^{-1} m$ equals half the total area of S). No solving for m is required, only the equation. Points: 1 for equating polar areas (one side equal to half the total area of S), 1 for using an inverse trigonometric function of m as a limit of integration, 1 for the complete correct equation.

Solution 2

(d)

Mark scheme

- As $k \to \infty$, the circle $r = k\cos\theta$ grows to enclose all points to the right of the y-axis, so the portion of S inside it eventually becomes all of S with $0 \leq \theta \leq \pi/2$.
  $\lim_{k\to\infty} A(k) = \frac{1}{2}\int_0^{\pi/2}(r(\theta))^2\,d\theta = \frac{1}{2}\int_0^{\pi/2}\left(3\sqrt{\theta}\sin(\theta^2)\right)^2\,d\theta = 3.324.$
  Points: 1 for the correct limits of integration (0 to $\pi/2$), 1 for the answer with the correct integral.

Question 5

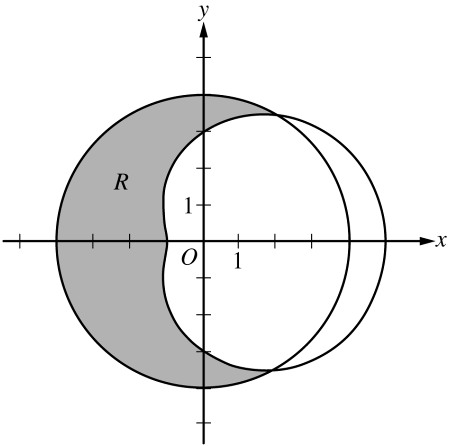
2018 ▪ Q5

[Figure: polar graph on x- and y-axes, showing two overlapping curves — the circle $r = 4$ (centered at the origin, radius 4) and the curve $r = 3 + 2\cos\theta$ (a limaçon shifted to the right). The two curves intersect, and the shaded region R is the crescent-shaped area inside the circle $r = 4$ and outside the limaçon $r = 3 + 2\cos\theta$, on the left side of the graph. Axis tick marks at 1 are labeled on both axes; origin labeled O.]

The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown above. The curves intersect at $\theta = \dfrac{\pi}{3}$ and $\theta = \dfrac{5\pi}{3}$.

(a) Let R be the shaded region that is inside the graph of $r = 4$ and also outside the graph of $r = 3 + 2\cos\theta$, as shown in the figure above. Write an expression involving an integral for the area of R.

Question 5



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The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown above. The curves intersect at $\theta = \dfrac{\pi}{3}$ and $\theta = \dfrac{5\pi}{3}$.

(b) Find the slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \dfrac{\pi}{2}$.

Question 5

2018 ▪ Q5

[Figure: polar graph on x- and y-axes, showing two overlapping curves — the circle $r = 4$ (centered at the origin, radius 4) and the curve $r = 3 + 2\cos\theta$ (a limaçon shifted to the right). The two curves intersect, and the shaded region R is the crescent-shaped area inside the circle $r = 4$ and outside the limaçon $r = 3 + 2\cos\theta$, on the left side of the graph. Axis tick marks at 1 are labeled on both axes; origin labeled O.]

The graphs of the polar curves $r = 4$ and $r = 3 + 2\cos\theta$ are shown above. The curves intersect at $\theta = \dfrac{\pi}{3}$ and $\theta = \dfrac{5\pi}{3}$.

(c) A particle moves along the portion of the curve $r = 3 + 2\cos\theta$ for $0 < \theta < \dfrac{\pi}{2}$. The particle moves in such a way that the distance between the particle and the origin increases at a constant rate of 3 units per second. Find the rate at which the angle θ

Question 5

changes with respect to time at the instant when the position of the particle corresponds to $\theta = \dfrac{\pi}{3}$. Indicate units of measure.

Solution 5

(a)

Mark scheme

- Area $= \dfrac{1}{2}\displaystyle\int_{\pi/3}^{5\pi/3} \left(4^2 - (3 + 2\cos\theta)^2\right) d\theta$. Points: 1 for the correct constant $(\frac{1}{2})$ and limits of integration ($\theta = \pi/3$ to $5\pi/3$, the intersection angles), 2 for the correct integrand $4^2 - (3 + 2\cos\theta)^2$ (outer curve squared minus inner curve squared).

Solution 5

(b) Mark scheme

- $\dfrac{dr}{d\theta} = -2\sin\theta \Rightarrow \left.\dfrac{dr}{d\theta}\right|_{\theta=\pi/2} = -2$, and $r(\pi/2) = 3 + 2\cos(\pi/2) = 3$. Since $y = r\sin\theta \Rightarrow \dfrac{dy}{d\theta} = \dfrac{dr}{d\theta}\sin\theta + r\cos\theta$ and $x = r\cos\theta \Rightarrow \dfrac{dx}{d\theta} = \dfrac{dr}{d\theta}\cos\theta - r\sin\theta$: $\left.\dfrac{dy}{dx}\right|_{\theta=\pi/2} = \left.\dfrac{(dy/d\theta)}{(dx/d\theta)}\right|_{\theta=\pi/2} = \dfrac{-2\sin(\pi/2) + 3\cos(\pi/2)}{-2\cos(\pi/2) - 3\sin(\pi/2)} = \dfrac{2}{3}$. The slope of the line tangent to the graph of $r = 3 + 2\cos\theta$ at $\theta = \pi/2$ is $\frac{2}{3}$. Points: 1 for the correct formula $\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$ (or equivalently $\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$), 1 for the correct $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ setup, 1 for the correct final answer $(2/3)$.

Solution 5

(c) Mark scheme

- $\dfrac{dr}{dt} = \dfrac{dr}{d\theta} \cdot \dfrac{d\theta}{dt} = -2\sin\theta \cdot \dfrac{d\theta}{dt} \Rightarrow \dfrac{d\theta}{dt} = \dfrac{dr}{dt} \cdot \dfrac{1}{-2\sin\theta}$. Given $\dfrac{dr}{dt} = 3$ (constant rate the distance from the origin increases), at $\theta = \pi/3$: $\left.\dfrac{d\theta}{dt}\right|_{\theta=\pi/3} = 3 \cdot \dfrac{1}{-2\sin(\pi/3)} = \dfrac{3}{-\sqrt{3}} = -\sqrt{3}$ radians per second. Points: 1 for relating $\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt}$, 1 for solving $\frac{d\theta}{dt} = \frac{dr}{dt} \cdot \frac{1}{-2\sin\theta}$, 1 for the correct final answer with units ($-\sqrt{3}$ radians per second).

Question 2

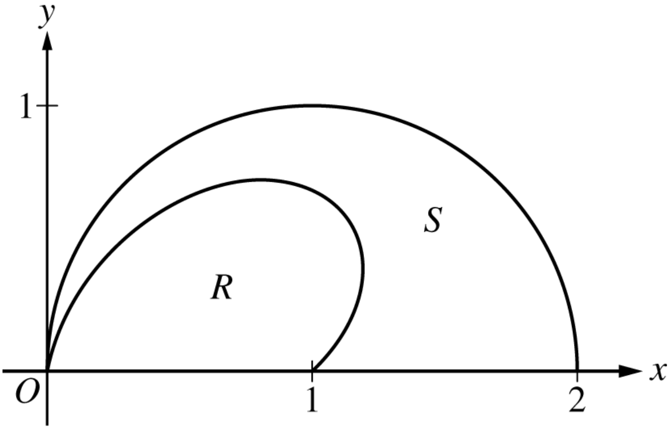
2017 ▪ Q2

[Graph in the xy-plane, first quadrant, x-axis from O to past 2 with tick marks at 1 and 2, y-axis with tick mark at 1. Two polar curves are shown, both starting and ending on the x-axis at O: a larger arc labeled S passing near/above $(1, 1)$ and returning to the x-axis at $x = 2$; a smaller inner loop labeled R lying under the larger arc, close to the origin, also returning to the x-axis near $x = 1$.]

The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \leq \theta \leq \dfrac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

Question 2



Question 2

2017 ▪ Q2

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(b) The ray $\theta = k$, where $0 < k < \dfrac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

Question 2

2017 ▪ Q2

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(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

Question 2

2017 ▪ Q2

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(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ? Give a reason for your answer.

Solution 2

(a)

Mark scheme

- Area of $R = \dfrac{1}{2}\int_0^{\pi/2} (f(\theta))^2 \, d\theta = 0.648414 \approx 0.648.$ 1 pt for the correct integral expression (polar area formula), 1 pt for the correct numeric answer 0.648.

Solution 2

(b)

Mark scheme

- An equation for k: $\displaystyle\int_0^k \left((g(\theta))^2 - (f(\theta))^2\right) d\theta = \frac{1}{2}\int_0^{\pi/2} \left((g(\theta))^2 - (f(\theta))^2\right) d\theta$ (equivalently $\int_0^k(\cdots)\,d\theta = \int_k^{\pi/2}(\cdots)\,d\theta$) — the integral from 0 to k of the area between g and f equals half the total area between g and f on $[0, \pi/2]$ (or equals the remaining integral from k to $\pi/2$). 1 pt for a correct integral expression representing the area of one region (between the curves, from 0 to k or k to $\pi/2$), 1 pt for setting it as a full equation whose solution gives k. Do NOT solve for k — only the equation is required.

Solution 2

(c)

Mark scheme

- $w(\theta) = g(\theta) - f(\theta)$. $w_A = \dfrac{\int_0^{\pi/2} w(\theta)\, d\theta}{\pi/2 - 0} = 0.485446 \approx 0.485$. 1 pt for the correct expression $w(\theta) = g(\theta) - f(\theta)$, 1 pt for the correct average-value integral setup, 1 pt for the correct numeric average value 0.485.

Solution 2

(d)

Mark scheme

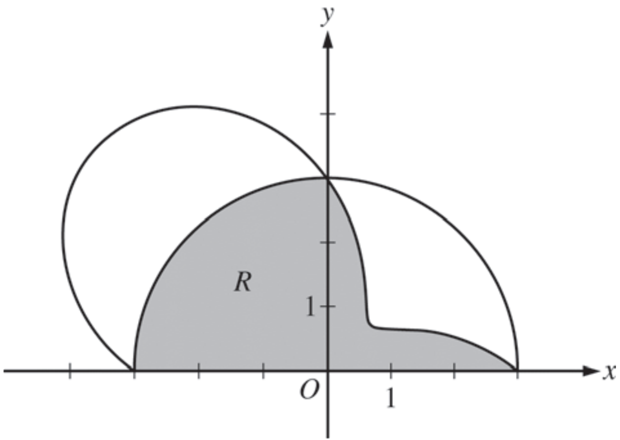
- Solving $w(\theta) = w_A$ for $0 \leq \theta \leq \pi/2$ gives $\theta = 0.517688 \approx 0.518$ (or 0.517). Since $w'(0.518) < 0$, $w(\theta)$ is decreasing at $\theta = 0.518$. 1 pt for correctly solving $w(\theta) = w_A$, 1 pt for the correct answer (decreasing) with a valid reason (sign of w' or equivalent, e.g. numerical/graphical justification).

Question 2

2014 ▪ Q2

[Graph showing two polar curves for $0 \leq \theta \leq \pi$: the circle $r = 3$ and the curve $r = 3 - 2\sin(2\theta)$, plotted on an xy-plane with axis labeled 1 near the origin O. The shaded region R, labeled near the point $(1, 1)$ roughly, is the region inside both curves — it is bounded above by the circle $r = 3$ arc and has a scalloped lower/right boundary from the curve $r = 3 - 2\sin(2\theta)$, meeting the x-axis near $x \approx 3$.]
The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.
(a) Let R be the shaded region that is inside the graph of $r = 3$ and inside the graph of $r = 3 - 2\sin(2\theta)$. Find the area of R.

Question 2



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2014 ▪ Q2

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The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

(b) For the curve $r = 3 - 2\sin(2\theta)$, find the value of $\dfrac{dx}{d\theta}$ at $\theta = \dfrac{\pi}{6}$.

Question 2

2014 ▪ Q2

[Graph showing two polar curves for $0 \leq \theta \leq \pi$: the circle $r = 3$ and the curve $r = 3 - 2\sin(2\theta)$, plotted on an xy-plane with axis labeled 1 near the origin O. The shaded region R, labeled near the point $(1, 1)$ roughly, is the region inside both curves — it is bounded above by the circle $r = 3$ arc and has a scalloped lower/right boundary from the curve $r = 3 - 2\sin(2\theta)$, meeting the x-axis near $x \approx 3$.]

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

(c) The distance between the two curves changes for $0 < \theta < \frac{\pi}{2}$. Find the rate at which the distance between the two curves is changing with respect to θ when $\theta = \frac{\pi}{3}$.

Question 2

2014 ▪ Q2

[Graph showing two polar curves for $0 \leq \theta \leq \pi$: the circle $r = 3$ and the curve $r = 3 - 2\sin(2\theta)$, plotted on an xy-plane with axis labeled 1 near the origin O. The shaded region R, labeled near the point $(1, 1)$ roughly, is the region inside both curves — it is bounded above by the circle $r = 3$ arc and has a scalloped lower/right boundary from the curve $r = 3 - 2\sin(2\theta)$, meeting the x-axis near $x \approx 3$.]

The graphs of the polar curves $r = 3$ and $r = 3 - 2\sin(2\theta)$ are shown in the figure above for $0 \leq \theta \leq \pi$.

(d) A particle is moving along the curve $r = 3 - 2\sin(2\theta)$ so that $\dfrac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\dfrac{dr}{dt}$ at $\theta = \dfrac{\pi}{6}$.