Past-paper walk-through

Integrating Vector-Valued Functions ▪ 9.5

eXercise

Questions in this set

- 2022 Q2
- 2015 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2022 ▪ Q2

A particle moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time $t > 0$. The particle moves in such a way that $\dfrac{dx}{dt} = \sqrt{1 + t^2}$ and $\dfrac{dy}{dt} = \ln(2 + t^2)$. At time $t = 4$, the particle is at the point $(1, 5)$.

(a) Find the slope of the line tangent to the path of the particle at time $t = 4$.

(b) Find the speed of the particle at time $t = 4$, and find the acceleration vector of the particle at time $t = 4$.

(c) Find the y-coordinate of the particle's position at time $t = 6$.

(d) Find the total distance the particle travels along the curve from time $t = 4$ to time $t = 6$.

Solution 2

(a) Mark scheme

- $\left.\dfrac{dy}{dx}\right|_{t=4} = \dfrac{y'(4)}{x'(4)} = \dfrac{\ln(2+4^2)}{\sqrt{1+4^2}} = \dfrac{\ln 18}{\sqrt{17}} = 0.701018 \approx 0.701$. 1 point for the correct numeric slope, with the setup (quotient of $y'(4)$ and $x'(4)$) evident in the response.

Solution 2

(b) Mark scheme

- Speed $= \sqrt{(x'(4))^2 + (y'(4))^2} = \sqrt{17 + (\ln 18)^2} = 5.035300 \approx 5.035$ (1 point).
  Acceleration: $x''(t) = \dfrac{t}{\sqrt{1+t^2}}$, $y''(t) = \dfrac{2t}{2+t^2}$, so
  $a(4) = \langle x''(4), y''(4) \rangle = \left\langle \dfrac{4}{\sqrt{17}}, \dfrac{4}{9} \right\rangle = \langle 0.970143, 0.444444 \rangle \approx \langle 0.970, 0.444 \rangle$ (1 point for each component, earned independently — if the components are reversed, neither point is earned).

Solution 2

(c)

Mark scheme

- $y(6) = y(4) + \int_4^6 \ln(2 + t^2)\, dt = 5 + 6.570517 = 11.570517 \approx 11.571$ (or 11.570). 3 points: 1 for the integrand $\ln(2 + t^2)$ appearing in an integral, 1 for adding the correct definite integral to $y(4) = 5$, 1 for the final numeric answer.

Solution 2

(d)

Mark scheme

- Total distance $= \int_4^6 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt =$ $\int_4^6 \sqrt{(1+t^2) + (\ln(2+t^2))^2}\, dt = 12.136228 \approx 12.136$. 2 points: 1 for the correct integrand in a definite integral from $t = 4$ to $t = 6$, 1 for the correct numeric answer (an unsupported answer of 12.136 earns neither point).

Question 2

2015 ▪ Q2

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos(t^2),\ e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

Solution 2

(a)

Mark scheme

- $x(2) = 3 + \int_1^2 \cos(t^2)\,dt = 2.557$ (or 2.556). Award 1 point for the correct integral setup, 1 point for using the initial condition $x(1) = 3$, and 1 point for the correct final answer.

Solution 2

(b)

Mark scheme

- $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^{0.5t}}{\cos(t^2)}$. Setting $\frac{e^{0.5t}}{\cos(t^2)} = 2$ gives $t = 0.840$. Award 1 point for the correct slope expression in terms of t, and 1 point for the correct answer.

Solution 2

(c)

Mark scheme

- Speed $= \sqrt{\cos^2(t^2) + e^t}$. Setting $\sqrt{\cos^2(t^2) + e^t} = 3$ gives $t = 2.196$ (or 2.195). Award 1 point for the correct speed expression in terms of t, and 1 point for the correct answer.

Solution 2

(d)

Mark scheme

- Distance $= \int_0^1 \sqrt{\cos^2(t^2) + e^t}\, dt = 1.595$ (or 1.594). Award 1 point for the correct integral setup, and 1 point for the correct numeric answer.