Past-paper walk-through

Volumes with Cross Sections: Squares and Rectangles ▪ 8.7

eXercise

Questions in this set

- 2022 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

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[Figure 1: Graph of $y = \frac{1}{x}$ in the first quadrant; x-axis marked at 1 and 5, curve decreasing from near the y-axis at $x = 1$ down toward the x-axis; the shaded region R is bounded by the curve, the x-axis, and the vertical lines $x = 1$ and $x = 5$, labelled "Figure 1".]

[Figure 2: Graph of $y = \frac{1}{x^2}$ in the first quadrant; x-axis marked at 3, curve decreasing from near the y-axis at $x = 3$ down toward the x-axis extending to the right unbounded; the region W is the unbounded region between the curve and the x-axis to the right of $x = 3$, labelled "Figure 2".]

Figures 1 and 2, shown above, illustrate regions in the first quadrant associated with the graphs of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$, respectively. In Figure 1, let R be the region bounded

Question 5

by the graph of $y = \dfrac{1}{x}$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. In Figure 2, let W be the unbounded region between the graph of $y = \dfrac{1}{x^2}$ and the x-axis that lies to the right of the vertical line $x = 3$.

(a) Find the area of region R.

(b) Region R is the base of a solid. For the solid, at each x the cross section perpendicular to the x-axis is a rectangle with area given by $xe^{x/5}$. Find the volume of the solid.

(c) Find the volume of the solid generated when the unbounded region W is revolved about the x-axis.

Question 5

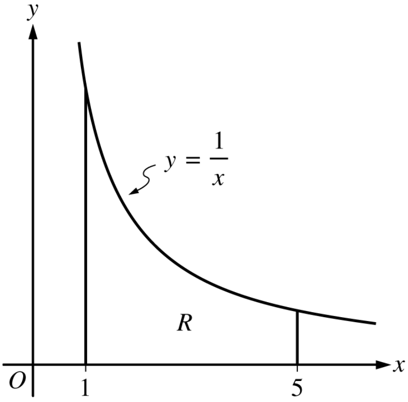

Figure 1

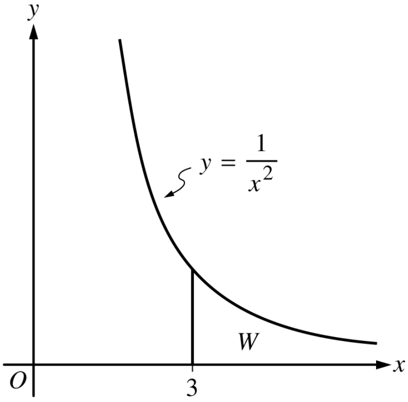

Figure 2

Solution 5

(a)

Mark scheme

- Area $= \int_1^5 \frac{1}{x}\,dx = \ln x\Big|_1^5 = \ln 5 - \ln 1 = \ln 5$. 2 points: 1 for the definite integral with correct limits ($x = 1$ to $x = 5$), 1 for the correct evaluated answer $\ln 5$.

Solution 5

(b) Mark scheme

- Volume $= \int_1^5 xe^{x/5}\,dx$. Integration by parts with $u = x$, $dv = e^{x/5}dx \Rightarrow du = dx$, $v = 5e^{x/5}$:
  $\int xe^{x/5}dx = 5xe^{x/5} - \int 5e^{x/5}dx = 5xe^{x/5} - 25e^{x/5} + C = 5e^{x/5}(x-5) + C$. Volume $= 5e^{x/5}(x-5)\Big|_1^5 = 5e^0(0) - 5e^{1/5}(-4) = 20e^{1/5}$. 4 points: 1 for the correct definite integral (a nonzero constant multiple is acceptable), 1 for correctly setting up integration by parts (u and dv), 1 for the antiderivative $5xe^{x/5} - \int 5e^{x/5}dx$ (or equivalent, e.g. tabular method), 1 for the final correct answer $20e^{1/5}$.

Solution 5

(c)

Mark scheme

- Volume $= \pi \int_3^\infty \left(\frac{1}{x^2}\right)^2 dx = \pi \lim_{b\to\infty} \int_3^b \frac{1}{x^4}\, dx = \pi \lim_{b\to\infty} \left[-\frac{1}{3x^3}\right]_3^b =$ $\pi \lim_{b\to\infty} \left(-\frac{1}{3}\right)\left[\frac{1}{b^3} - \frac{1}{27}\right] = \pi\left(-\frac{1}{3}\right)\left(0 - \frac{1}{27}\right) = \frac{\pi}{81}$. 3 points: 1 for setting up the improper integral as a limit (a nonzero constant multiple is acceptable), 1 for a correct antiderivative of the form $1/x^n$ ($n \geq 2$ integer), 1 for the final correct answer $\pi/81$ using correct limit notation (no arithmetic with infinity).