Past-paper walk-through

Connecting Position, Velocity, and Acceleration of Functions Using Integrals ▪ 8.2

eXercise

Questions in this set

- 2015 Q3
- 2014 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2015 ▪ Q3

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v. Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

(a) Use the data in the table to estimate the value of $v'(16)$.

(b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)|\, dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)|\, dt$ using a right Riemann sum with the four subintervals indicated in the table.

Question 3

(c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

(d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	–220	150

Solution 3

(a)

Mark scheme

- $v'(16) \approx \frac{240-200}{20-12} = 5$ meters/min^2, using the difference quotient with the table values at $t = 12$ and $t = 20$ (which bracket $t = 16$). Award 1 point for a reasonable approximation using appropriate table values.

Solution 3

(b)

Mark scheme

- $\int_0^{40} |v(t)|\, dt$ represents the total distance Johanna jogs, in meters, over the time interval $0 \le t \le 40$ minutes. Using a right Riemann sum with the four given subintervals: $\int_0^{40} |v(t)|\, dt \approx 12|v(12)| + 8|v(20)| + 4|v(24)| + 16|v(40)| = 12(200) + 8(240) + 4(220) + 16(150) = 2400 + 1920 + 880 + 2400 = 7600$ meters. Award 1 point for the correct explanation (total distance in meters), 1 point for correct right-Riemann-sum setup with the four given subintervals, and 1 point for the correct approximation (7600 meters).

Solution 3

(c)

Mark scheme

- Bob's acceleration is $B'(t) = 3t^2 - 12t$. $B'(5) = 3(25) - 12(5) = 15$ meters/min^2. Award 1 point for using $B'(t)$ (the derivative of the velocity model), and 1 point for the correct answer.

Solution 3

(d) Mark scheme

- Average velocity $= \frac{1}{10}\int_0^{10}(t^3 - 6t^2 + 300)\,dt = \frac{1}{10}\left[\frac{t^4}{4} - 2t^3 + 300t\right]_0^{10} = \frac{1}{10}\left[\frac{10000}{4} - 2000 + 3000\right] = 350$ meters/min. Award 1 point for the correct integral setup, 1 point for the correct antiderivative, and 1 point for the correct answer.

Question 4

2014 ▪ Q4

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t **(minutes)**	**0**	**2**	**5**	**8**	**12**
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

Question 4

(a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.

Question 4

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	–120	–150