Past-paper walk-through

Finding the Average Value of a Function on an Interval ▪ 8.1

eXercise

Questions in this set

- 2026 Q2
- 2025 Q1
- 2024 Q1
- 2023 Q1
- 2022 Q1
- 2021 Q1
- 2019 Q1
- 2019 Q2
- 2018 Q4
- 2017 Q2

Questions in this set

- 2016 Q5
- 2015 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 2

2026 ▪ Q2

Let S be the shaded region bounded by the graph of the polar curve

$$r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta) \text{ for } 0 \leq \theta \leq \pi,$$

as shown in the figure.

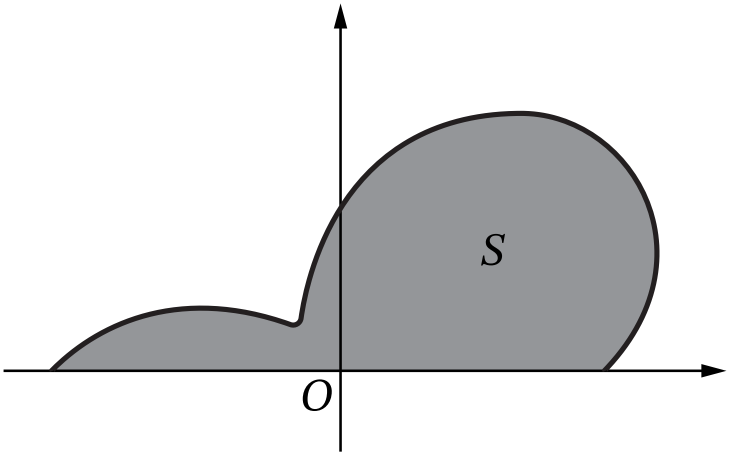
[Graph in the xy-plane showing a polar curve traced for $0 \leq \theta \leq \pi$. The curve produces two lobes above the x-axis: a smaller bump straddling the origin O to the left of the y-axis and dipping slightly below near the y-axis, and a larger shaded lobe labeled S to the right of the y-axis, bulging out to about $x = 3$ and reaching up to about $y = 3$ before returning to the x-axis around $x \approx 4$. The y-axis is marked with a tick at 1.]

It can be shown that $r'(\theta) = 4\cos(2\theta) - 2\sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

(a) Find the area of region S. Show the setup for your calculations.

Question 2



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2026 ▪ Q2

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Question 2

(b) There is a point on the curve at which the slope of the line tangent to the curve is $-\frac{3}{7}$. At this point, $\frac{dy}{d\theta} = \frac{3\sqrt{2}}{2}$. Find $\frac{dx}{d\theta}$ at this point. Show the work that leads to your answer.

Question 2

2026 ▪ Q2

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It can be shown that $r'(\theta) = 4\cos(2\theta) - 2\sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

(c)(i) Find the value of θ in the interval $0 < \theta < \dfrac{\pi}{2}$ at which r has a critical point.

Question 2

(c)(ii) Use a derivative test to determine whether the critical point is the location of a relative minimum, a relative maximum, or neither for r.

Question 2

2026 ▪ Q2

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It can be shown that $r'(\theta) = 4\cos(2\theta) - 2\sin(2\theta)$.

(Note: Your calculator should be in radian mode.)

Question 2

(d) Find the average distance from the origin to a point on the polar curve

$$r(\theta) = 3 + 2\sin(2\theta) + \cos(2\theta) \text{ for } \frac{\pi}{2} \leq \theta \leq \pi.$$

Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$. (Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \displaystyle\int_4^t 0.1 \cdot \ln(x)\, dx$, models the number of acres affected by the species over the time interval $4 \leq t \leq 36$. At what time t, for $4 \leq t \leq 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a)

Mark scheme

- Average value $= \dfrac{1}{4-0}\int_0^4 C(t)\,dt$. Since $\int_0^4 C(t)\,dt = 11.112896$, the average $= \dfrac{1}{4}(11.112896) = 2.778224 \approx 2.778$ acres. (1 pt: correct integral/average-value setup with division by 4; 1 pt: correct answer, accurate to 3 decimal places.)

Solution 1

(b)

Mark scheme

- Average rate of change of C on $[0,4]$: $\dfrac{C(4)-C(0)}{4-0} = 1.282008$. Set $C'(t) = \dfrac{38}{25+t^2}$ equal to this and solve: $\dfrac{38}{25+t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$. (1 pt: uses/presents the average rate of change; 1 pt: correct answer with supporting work.)

Solution 1

(c)

Mark scheme

- $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \frac{38}{25+t^2} = 0$. (1 pt: correct limit expression $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$; 1 pt: correct value, 0.)

Solution 1

(d)

Mark scheme

- $A(t) = C(t) - \int_4^t 0.1 \ln x \, dx \Rightarrow A'(t) = C'(t) - 0.1 \ln t$. Setting $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test on $[4, 36]$: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum occurs at $t = 11.442$ (or 11.441) weeks. (1 pt: considers $A'(t) = 0$; 1 pt: justification via global/candidates-test argument; 1 pt: correct answer with supporting work.)

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \leq t \leq 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \leq t \leq 7$. Show the work that leads to your answer and include units of measure.

Question 1

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \dfrac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

Question 1

2024 ▪ Q1

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100-t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Solution 1

(a)

Mark scheme

- C'(5) $\approx \dfrac{C(7)-C(3)}{7-3} = \dfrac{69-85}{4} = -4 degreesCelsiusperminute.Points: 1 foradifference-quotientsetup(anumeratordifferenceoveradenominatordifference, e.g.(69-85)/(7-3)); 1 forthecorrectunits(degreesCelsiusperminute) attachedtoanumericalvalue.$

Solution 1

(b) Mark scheme

- Left Riemann sum with subintervals [0,3],[3,7],[7,12]: $\int_0^{12} C(t)\,dt \approx (3-0)C(0)+(7-3)C(3)+(12-7)C(7) = 3(100)+4(85)+5(69) = 985. Interpretation : \frac{1}{12}\int_0^{12} C(t)\,dtistheaveragetemperatureofthecoffee(indegreesCelsius)overthetimeintervalfromt =$ $0tot = 12. Points :$ $1forthecorrectformoftheleftRiemannsum(correctsubintervalwidthstimesleft -$ $endpointvalues), 1forthenumericestimate985, 1fortheinterpretationreferencingboth' averagetem$

Solution 1

(c)

Mark scheme

- $C(20) = C(12) + \int_{12}^{20} C'(t)\,dt = 55 + (-14.670812) = 40.329188 \approx 40.329 (or 40.328) degrees Celsius. Points : 1 for a definite integral of C'(t) with limits 12 to 20, 1 for adding the initial condition C(12) = 55 to that integral, 1 for the correct final numeric answer 40.329 (or 40.328), with supporting work.$

Solution 1

(d)

Mark scheme

- Because C"(t) $= \dfrac{0.2455e^{0.01t}(100-t)}{t^2} > 0for12 < t < 20(sincet < 100there), C'(t)isincreasingonthisinterval, sothetemperatureofthecoffeeischangingatanincreas$

Question 1

2023 ▪ Q1

A customer at a gas station is pumping gasoline into a gas tank. The rate of flow of gasoline is modeled by a differentiable function f, where $f(t)$ is measured in gallons per second and t is measured in seconds since pumping began. Selected values of $f(t)$ are given in the table.

t **(seconds)**	**0**	**60**	**90**	**120**	**135**	**150**
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0

Question 1

(a) Using correct units, interpret the meaning of $\int_{60}^{135} f(t)\,dt$ in the context of the problem. Use a right Riemann sum with the three subintervals $[60, 90]$, $[90, 120]$, and $[120, 135]$ to approximate the value of $\int_{60}^{135} f(t)\,dt$.

Question 1

t (seconds)	0	60	90	120	135	150
$f(t)$ (gallons per second)	0	0.1	0.15	0.1	0.05	0