Past-paper walk-through

The Arc Length of a Smooth, Planar Curve and Distance Traveled ▪ 8.13

eXercise

Questions in this set

- 2026 Q5
- 2024 Q5
- 2014 Q5

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

2026 ▪ Q5

Let f be the function defined by $f(x) = \sqrt[3]{x-1}$, and let g be the function defined by $g(x) = e^{(-2x+4)}$. The graphs of f and g intersect at the point $(2, 1)$. Let R be the region bounded by the graph of f, the x-axis, and the vertical line $x = 2$, as shown in the figure.

[Graph in the xy-plane: y-axis marked at 1, x-axis marked at 1, 2, 3. The curve $y = f(x)$ starts near $x = 1$ (where it crosses the x-axis) and rises to the right, passing through $(2, 1)$; the curve $y = g(x)$ decreases from upper left, also passing through $(2, 1)$, and continues decreasing toward the right past $x = 3$. The shaded region R is bounded below by the x-axis, on the left by the curve $y = f(x)$, and on the right by the vertical line $x = 2$, lying between $x = 1$ and $x = 2$.]

(a) Evaluate $\displaystyle\int_1^2 f(x)\,dx$. Show the work that leads to your answer.

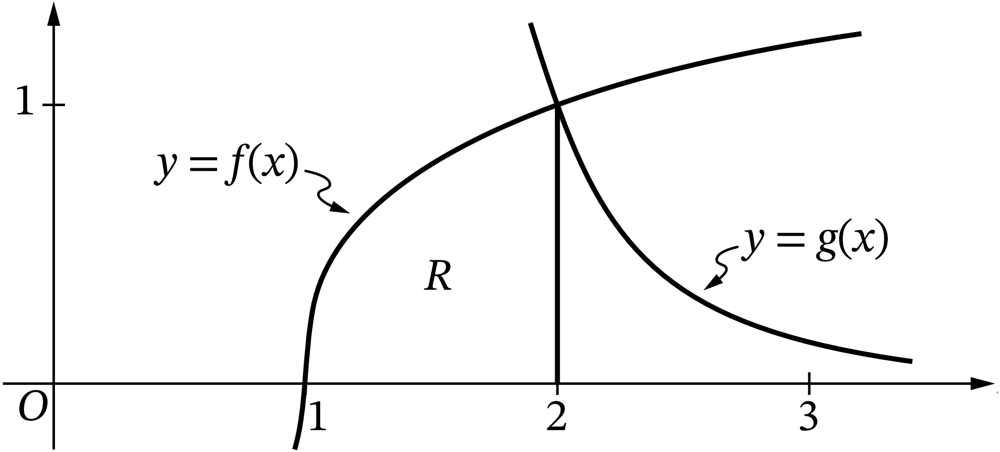
Question 5

(b) Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid generated when region R is rotated about the x-axis.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R.

(d) Evaluate $\displaystyle\int_2^{\infty} g(x)\,dx$. Show the work that leads to your answer.

Question 5



Question 5

2024 ▪ Q5

x	$\mathbf{0}$	π	2π
$f'(x)$	5	6	0

The function f is twice differentiable for all x with $f(0) = 0$. Values of f', the derivative of f, are given in the table for selected values of x.

Question 5

(a) For $x \geq 0$, the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$. Find the value of $h'(\pi)$. Show the work that leads to your answer.

(b) What information does $\int_0^\pi \sqrt{1 + (f'(x))^2}\, dx$ provide about the graph of f?

(c) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(2\pi)$. Show the computations that lead to your answer.

(d) Find $\int (t+5)\cos\left(\frac{t}{4}\right)\, dt$. Show the work that leads to your answer.

Question 5

x	0	π	2π
$f'(x)$	5	6	0

Solution 5

(a) Mark scheme

- For x$\geq 0, h(x) = \int_0^x \sqrt{1+(f'(t))^2}\,dt, sobytheFundamentalTheoremofCalculush'(x) = \sqrt{1+(f'(x))^2}. Usingf'(\pi) = 6fromthegiventable, h'(\pi) = \sqrt{1+6^2} = \sqrt{37}. Points : 1forthecorrectFTCsetuph'(x) = \sqrt{1+(f'(x))^2}(or\sqrt{1+(f'(\pi))^2}), 1forthecorrectfinalanswer\sqrt{37}.$

Solution 5

(b)

Mark scheme

- $\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx is the arc length of the graph of f on the interval [0, \pi]. Points : 1 for identifying 'arc length' (or an equivalent phrase such as 'distance along the curve') explicitly conne$

Solution 5

(c)

Mark scheme

- Using Euler's method with step size $\pi, startingatx = 0 : f(\pi) \approx f(0) + \pi f'(0) = 0 + 5\pi = 5\pi(usingf(0) = 0andf'(0) = 5fromthetable). Thenf(2\pi) \approx f(\pi) + \pi f'(\pi) \approx 5\pi + \pi(6) = 5\pi + 6\pi = 11\pi(usingf'(\pi) = 6fromthetable).Points : 1fordemonstratingtwoEuler's - methodstepsusingthecorrectexpressionfordy/dx(values5and6fromthetable)withatmostoneerro$
  $11\pi(bothpointsrequirethecorrecttablevalues5\pi and6\pi tobecombinedcorrectly).$

Solution 5

(d) Mark scheme

- Integration by parts with u=t+5, dv=$\cos(t/4)\,dt, sodu = dt, v = 4\sin(t/4) : \int (t+5)\cos\left(\frac{t}{4}\right)\,dt = 4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right)\,dt = 4(t+5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C.Points : 1 forcorrectuanddv(withcorrectduandv, possiblyimpliedviathetabularmethod), 1 forthecorrectforr$
  $\int v\,du, i.e. 4(t+5)\sin(t/4) - \int 4\sin(t/4)\,dt(oramathematicallyequivalentexpression), 1 forthecorrectfinalanswer 4(t+5)\sin(t/4) + 16\cos(t/4) + C(equivalentformssuchas 4t\sin(t/4) + 20\sin(t/4) + 16\cos(t/4) + Calsoearnthispoint), includingtheconstantofintegration.$

Question 5

2014 ▪ Q5

[Graph showing the shaded region R in the first quadrant (and dipping slightly below the x-axis) bounded by the curve $y = xe^{x^2}$ (rising steeply and concave up from the origin O through $x = 1$), the line $y = -2x$ (a straight line from the origin through the fourth quadrant), and the vertical line $x = 1$; axis labels O at the origin, 1 marked on the x-axis, x and y axis arrows shown; region R labeled inside the shaded area.]

Let R be the shaded region bounded by the graph of $y = xe^{x^2}$, the line $y = -2x$, and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of R.

Question 5