Past-paper walk-through

Finding Particular Solutions Using Initial Conditions and Separation of Variables ▪ 7.7

eXercise

Questions in this set

- 2026 Q3
- 2024 Q3
- 2023 Q3
- 2021 Q5
- 2019 Q4
- 2017 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation

$$\frac{dH}{dt} = -\frac{1}{15}(H - 20),$$

where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

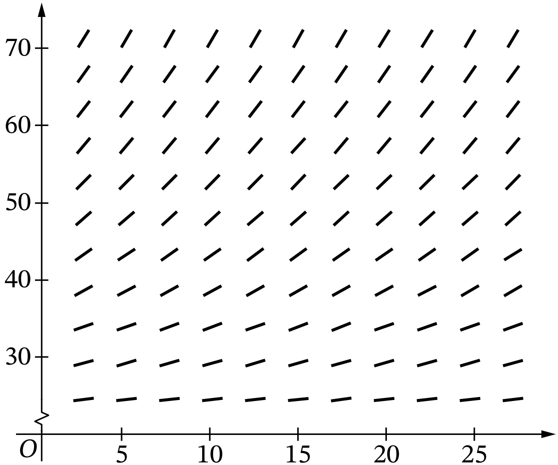
(a) Explain why the following could not be a slope field for the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{15}(H - 20)$.

[Slope field graph: horizontal axis t from 0 to about 28 (gridlines at 5, 10, 15, 20, 25), vertical axis H from about 25 to 70 (gridlines at 30, 40, 50, 60, 70). Short line

Question 3

segments are drawn at each grid point. Near the top of the grid (around $H = 70$) the segments are steep, close to vertical, positive slope; moving down the grid the segments rotate, becoming shallow near $H = 30$, with negative slope by the bottom rows. At a given height H, the segments look the same regardless of t (slopes vary only with H, not with t, matching the autonomous equation) — but the segments are shown with a mix of orientations inconsistent with the required sign pattern of $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.]

Question 3



Question 3

2026 ▪ Q3

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where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(b) Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

Question 3

2026 ▪ Q3

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$$\frac{dH}{dt} = -\frac{1}{15}(H - 20),$$

where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(c) It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

Question 3

2026 ▪ Q3

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$$\frac{dH}{dt} = -\frac{1}{15}(H - 20),$$

where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(d) Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

Question 3

2024 ▪ Q3

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = \dfrac{1}{2}(H-1)\cos\left(\dfrac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon $(t = 0)$. It is known that $H(0) = 4$.

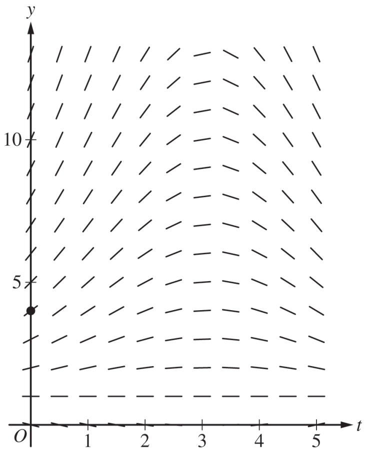
(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.

[Slope field graph; t-axis (horizontal) from 0 to 5 with gridlines at 1, 2, 3, 4, 5; y-axis (vertical) with gridlines at 5 and 10. A dot marks the point $(0, 4)$ on the y-axis. The slope field shows short line segments at grid points across the region $0 \le t \le 5$: for small y (near the t-axis) the segments are nearly horizontal (flat dashes); moving up toward $y = 5$–10, the segments tilt, transitioning from forward slashes ($/$, positive slope) on the left side of the grid (small t) through nearly vertical/horizontal ($-$ or $\sim$)

Question 3

in the middle, to backslashes ($\backslash$, negative slope) on the right side (larger t), reflecting the $\cos(t/2)$ factor changing sign.]

Question 3



Question 3

2024 ▪ Q3

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(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Question 3

2024 ▪ Q3

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon $(t=0)$. It is known that $H(0)=4$.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0)=4$.

Solution 3

(a)

Mark scheme

- This is a graphical sketching item: the solution curve y=H(t) must be drawn through the point (0,4), rising to a local maximum near t=$\pi(visuallyaroundH \approx 9 - 10onthegivenslopefield), thendecreasing, andmustextendtoatleasttt = 4.5withnoobviousconflictswiththegivenslope-fieldsegments.Thesinglepointisearnedonlyforah$
$drawncurvesatisfyingthesegraphicalrequirements; thereisnoseparatenumericoralgebraicanswer$

Solution 3

(b) Mark scheme

- Since H(t)>1 for 0<t<5,
  $\dfrac{dH}{dt} = \dfrac{1}{2}(H-1)\cos\left(\dfrac{t}{2}\right) = 0 requires \cos(t/2) = 0, whichgivesthecriticalpointt = \pi. For 0 < t < \pi, dH/dt > 0, andfor \pi < t < 5, dH/dt < 0 (since H-1 > 0 throughoutwhile \cos(t/2) changessignfrompositivetonegativeatt = \pi); thereforet = \pi isthelocationofarelativemaximumofH(equivalently, asecond - derivativetestatt = \pi givesd^2 H/dt^2 < 0). Points: 1 forconsideringthesignofdH/dt(equivalentlydH/dt = 0, dH/dt > 0, dH/dt < 0, \cos(t/2) = 0, or \cos(t/2) > 0/ < 0), 1 foridentifyingt = \pi asthecriticalpoint(withorwithoutsupportingwork), 1 forthecorrectclassification' relativemaximu$
  $\pi, oranequivalentsecond - derivativetest); thisthirdpointcannotbeearnedwithoutthefirst.$

Solution 3

(c) Mark scheme

- Separating variables: $\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right) dt. Integrating both sides: \int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right) dt \;\Rightarrow\; \ln|H-1| = \sin\left(\frac{t}{2}\right) + C. Using H(0) = 4: \ln|4-1| = \sin(0) + C \Rightarrow C = \ln 3. Since H(0) = 4 > 1, |H-1| = H-1, so \ln(H-1) = \sin(t/2) + \ln 3 \Rightarrow H-1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)} \Rightarrow H(t) = 1 + 3e^{\sin(t/2)}. Points (5 total, sequential): 1 for correct separation of variables, 1 for one correct antiderivative (either side), 1 for the second correct$ $4 (eligible only if the separation point and at least one antiderivative point were earned), 1 for the correct f$ $form answer H(t) = 1 + 3e^{\sin(t/2)} (eligible only if all first four points were earned).$

Question 3

2023 ▪ Q3

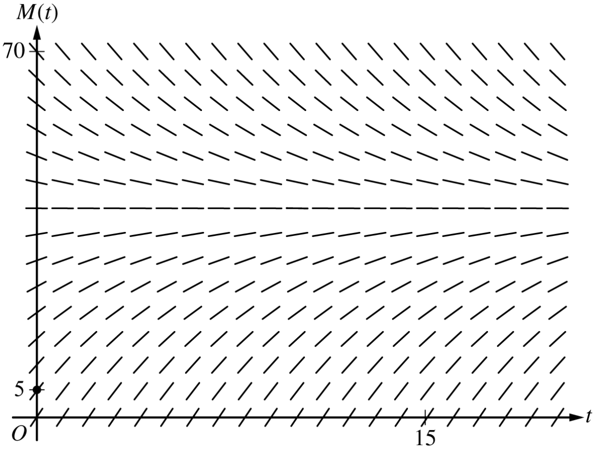
A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.
[Slope field diagram: axes labeled $M(t)$ (vertical) and t (horizontal); vertical axis marked at 5 and 70, horizontal axis marked at 15. Short line segments fill the plane:

Question 3

below $M = 40$ the segments have positive slope (steeper near $M = 5$, shallower approaching $M = 40$); at $M = 40$ the segments are horizontal (slope 0); above $M = 40$ the segments have negative slope. The point $(0, 5)$ is marked on the vertical axis.]

Question 3



Question 3

2023 ▪ Q3

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(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

Question 3

2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

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2023 ▪ Q3

A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius ($^\circ$C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\dfrac{dM}{dt} = \dfrac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

Solution 3

(a) Mark scheme

- The solution curve to $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ through $(0, 5)$ starts at $(0, 5)$, rises with decreasing steepness (concave down, since $M < 40$), and approaches the horizontal asymptote $M = 40$ from below as t increases, extending close to the right edge of the given rectangle and staying entirely below the $M = 40$ line, consistent with the slope field. (1 point for a curve passing through $(0, 5)$, extending reasonably close to the left/right edges of the rectangle, with no obvious conflicts with the given slope segments, lying entirely below $M = 40$.)

Solution 3

(b) Mark scheme

- $\left.\frac{dM}{dt}\right|_{t=0} = \frac{1}{4}(40-5) = \frac{35}{4}$. Tangent line: $y = 5 + \frac{35}{4}(t-0)$, so $M(2) \approx 5 + \frac{35}{4} \cdot 2 = 22.5$. The temperature of the milk at $t = 2$ minutes is approximately $22.5°$C. (1 point for the correct value $\frac{35}{4}$ of $\frac{dM}{dt}$ at $t = 0$; 1 point for a supported approximation using a tangent line through $(0,5)$ with slope $\frac{35}{4}$ — an unsupported approximation does not earn this point.)

Solution 3

(c) Mark scheme

- $\frac{d^2M}{dt^2} = -\frac{1}{4}\frac{dM}{dt} = -\frac{1}{4}\left(\frac{1}{4}(40 - M)\right) = -\frac{1}{16}(40 - M)$. Since $M(t) < 40$, $\frac{d^2M}{dt^2} < 0$, so the graph of M is concave down; therefore the tangent-line approximation from part (b) is an overestimate. (1 point for the correct expression $\frac{d^2M}{dt^2} = -\frac{1}{16}(40 - M)$, or equivalent in terms of $\frac{dM}{dt}$; 1 point for concluding 'overestimate' with reasoning based on $\frac{d^2M}{dt^2} < 0$/concave down over the relevant interval — an argument based on concavity at a single point does not earn this point.)

Solution 3

(d) Mark scheme

- Separate variables: $\frac{dM}{40-M} = \frac{1}{4}\,dt$, so $\int \frac{dM}{40-M} = \int \frac{1}{4}\,dt$, giving $-\ln|40-M| = \frac{1}{4}t + C$. Using $M(0)=5$: $-\ln|40-5| = 0 + C \Rightarrow C = -\ln 35$. Since $M(t) < 40$, $40 - M > 0$ so $|40-M| = 40-M$: $-\ln(40-M) = \frac{1}{4}t - \ln 35$, i.e. $\ln(40-M) = -\frac{1}{4}t + \ln 35$, so $40 - M = 35e^{-t/4}$, giving $M(t) = 40 - 35e^{-t/4}$. (1 point for separating variables; 1 point for finding antiderivatives $-\ln|40-M|$ and $\frac{1}{4}t$; 1 point for including the constant of integration and correctly substituting the initial condition $M(0)=5$; 1 point — only earned if all prior points are earned — for solving to get $M(t) = 40 - 35e^{-t/4}$ or an equivalent form.)

Question 5

2021 ▪ Q5

Let $y = f(x)$ be the particular solution to the differential equation $\dfrac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$. It can be shown that $f''(1) = 4$.

(a) Write the second-degree Taylor polynomial for f about $x = 1$. Use the Taylor polynomial to approximate $f(2)$.

(b) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(2)$. Show the work that leads to your answer.

(c) Find the particular solution $y = f(x)$ to the differential equation $\dfrac{dy}{dx} = y \cdot (x \ln x)$ with initial condition $f(1) = 4$.

Solution 5

(a)

Mark scheme

- $f'(1) = \left.\dfrac{dy}{dx}\right|_{(1,4)} = 4 \cdot (1 \ln 1) = 0$ (and it is given that $f''(1) = 4$).
  Second-degree Taylor polynomial about $x = 1$:
  $P(x) = f(1) + f'(1)(x-1) + \dfrac{f''(1)}{2!}(x-1)^2 = 4 + 0(x-1) + 2(x-1)^2$. Using it: $f(2) \approx 4 + 2(2-1)^2 = 6$. (1 pt: correct Taylor polynomial $4 + 2(x-1)^2$ [or equivalent, must be centered at $x = 1$]; 1 pt: approximation $f(2) \approx 6$.)

Solution 5

(b)

Mark scheme

- Euler's method with $\Delta x = 0.5$ starting at $(1, 4)$: Step 1: $f(1.5) \approx f(1) + 0.5 \cdot \left.\dfrac{dy}{dx}\right|_{(1,4)} = 4 + 0.5 \cdot [4 \cdot (1 \ln 1)] = 4 + 0 = 4$. Step 2: $f(2) \approx f(1.5) + 0.5 \cdot \left.\dfrac{dy}{dx}\right|_{(1.5,4)} = 4 + 0.5 \cdot [4 \cdot (1.5 \ln 1.5)] = 4 + 3 \ln 1.5$ (≈ 5.216). (1 pt: two correct Euler steps of size 0.5 using $dy/dx = y(x \ln x)$ and $f(1) = 4$, at most one error; 1 pt: final answer $4 + 3 \ln 1.5$ — requires the first point.)

Solution 5

(c) Mark scheme

- Separate variables: $\dfrac{1}{y}\,dy = x\ln x\,dx$. Integrate the right side by parts ($u = \ln x$, $dv = x\,dx$): $\int x\ln x\,dx = \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} + C$. Left side: $\int \dfrac{1}{y}\,dy = \ln|y|$. So $\ln|y| = \dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} + C$. Apply $f(1) = 4$: $\ln 4 = 0 - \dfrac{1}{4} + C \Rightarrow C = \ln 4 + \dfrac{1}{4}$. Solve for y: $y = e^{\frac{x^2}{2}\ln x - \frac{x^2}{4} + \ln 4 + \frac{1}{4}}$, valid for $x > 0$. (1 pt: correctly separates variables; 1 pt: correct antiderivative of $x\ln x$ via integration by parts; 1 pt: correct antiderivative $\ln|y|$ for $\frac{1}{y}$; 1 pt: correctly finds the constant of integration

Solution 5

using $f(1) = 4$ — requires the separation point and at least one antiderivative point; 1 pt: solves explicitly for y — requires all four prior points.)

Question 4

2019 ▪ Q4

[Figure: A vertical cylindrical barrel with diameter labeled "2 ft" across the top. The barrel is shaded (filled with water) from the bottom up to a level marked with a bracket labeled "h ft", with an unshaded empty region above the water level up to the rim.]

A cylindrical barrel with a diameter of 2 feet contains collected rainwater, as shown in the figure above. The water drains out through a valve (not shown) at the bottom of the barrel. The rate of change of the height h of the water in the barrel with respect to time t is modeled by $\frac{dh}{dt} = -\frac{1}{10}\sqrt{h}$, where h is measured in feet and t is measured in seconds. (The volume V of a cylinder with radius r and height h is $V = \pi r^2 h$.)

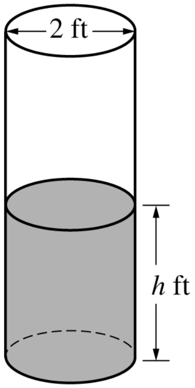
(a) Find the rate of change of the volume of water in the barrel with respect to time when the height of the water is 4 feet. Indicate units of measure.

Question 4

(b) When the height of the water is 3 feet, is the rate of change of the height of the water with respect to time increasing or decreasing? Explain your reasoning.

(c) At time $t = 0$ seconds, the height of the water is 5 feet. Use separation of variables to find an expression for h in terms of t.

Question 4



Solution 4

(a) Mark scheme

- The barrel is a cylinder of radius 1 ft, so $V = \pi r^2 h = \pi(1)^2 h = \pi h$. Given $\dfrac{dh}{dt} = -\dfrac{1}{10}\sqrt{h}$, $\dfrac{dV}{dt}\Big|_{h=4} = \pi \dfrac{dh}{dt}\Big|_{h=4} = \pi\left(-\dfrac{1}{10}\sqrt{4}\right) = -\dfrac{\pi}{5}$ cubic feet per second. Points: 1 for $\dfrac{dV}{dt} = \pi\dfrac{dh}{dt}$, 1 for the answer with correct units (cubic feet per second).

Solution 4

(b) Mark scheme

- $\dfrac{d}{dh}\left(-\dfrac{1}{10}\sqrt{h}\right) = -\dfrac{1}{20\sqrt{h}}$. By the chain rule, $\dfrac{d^2h}{dt^2} = -\dfrac{1}{20\sqrt{h}} \cdot \dfrac{dh}{dt} = -\dfrac{1}{20\sqrt{h}} \cdot \left(-\dfrac{1}{10}\sqrt{h}\right) = \dfrac{1}{200}$. Because $\dfrac{d^2h}{dt^2} = \dfrac{1}{200} > 0$ for $h > 0$, the rate of change of the height of the water is increasing when the height of the water is 3 feet. Points: 1 for $\dfrac{d}{dh}\left(-\dfrac{1}{10}\sqrt{h}\right) = -\dfrac{1}{20\sqrt{h}}$, 1 for $\dfrac{d^2h}{dt^2} = -\dfrac{1}{20\sqrt{h}} \cdot \dfrac{dh}{dt}$, 1 for the answer with explanation.

Solution 4

(c) Mark scheme

- Separate variables: $\dfrac{dh}{\sqrt{h}} = -\dfrac{1}{10}\,dt$. Integrate both sides: $\displaystyle\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{10}\,dt \Rightarrow 2\sqrt{h} = -\frac{1}{10}t + C$. Apply the initial condition $h(0) = 5$: $2\sqrt{5} = -\dfrac{1}{10}(0) + C \Rightarrow C = 2\sqrt{5}$. So $2\sqrt{h} = -\dfrac{1}{10}t + 2\sqrt{5}$, giving $h(t) = \left(-\dfrac{1}{20}t + \sqrt{5}\right)^2$. Points: 1 for separation of variables, 1 for antiderivatives, 1 for the constant of integration found using the initial condition, 1 for the final expression $h(t)$. Note: award $0/4$ if no separation of variables is

Solution 4

shown; award at most 2/4 if separation of variables and antiderivatives are correct but no constant of integration is found.

Question 4

2017 ▪ Q4

At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ($^\circ$C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = -\dfrac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\dfrac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

Question 4

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\dfrac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

Solution 4

(a)

Mark scheme

- $H'(0) = -\frac{1}{4}(91 - 27) = -16$; $H(0) = 91$. Tangent line: $y = 91 - 16t$. Approximate internal temperature at $t = 3$: $91 - 16(3) = 43$ degrees Celsius. 1 pt for the correct slope -16, 1 pt for the correct tangent-line equation $y = 91 - 16t$, 1 pt for the correct approximation 43 (degrees Celsius).

Solution 4

(b)

Mark scheme

- $\dfrac{d^2H}{dt^2} = -\dfrac{1}{4}\dfrac{dH}{dt} = \left(-\tfrac{1}{4}\right)\left(-\tfrac{1}{4}\right)(H-27) = \dfrac{1}{16}(H-27)$. Since $H > 27$ for $t > 0$, $\dfrac{d^2H}{dt^2} > 0$ for $t > 0$, so the graph of H is concave up for $t > 0$. Thus the tangent-line approximation in part (a) is an underestimate. 1 pt for the correct conclusion (underestimate) with this concavity-based reasoning.

Solution 4

(c) Mark scheme

- Separating variables:
  $$\frac{dG}{(G-27)^{2/3}} = -dt \Rightarrow \int \frac{dG}{(G-27)^{2/3}} = \int (-1)\, dt \Rightarrow 3(G-27)^{1/3} = -t + C.$$
  Using $G(0) = 91$: $3(91-27)^{1/3} = 0 + C \Rightarrow C = 12$. So $3(G-27)^{1/3} = 12 - t$, giving $G(t) = 27 + \left(\frac{12-t}{3}\right)^3$ for $0 \leq t < 10$. At $t = 3$: $G(3) = 27 + \left(\frac{12-3}{3}\right)^3 = 27 + 27 = 54$ degrees Celsius. 5 pts total: 1 for separating variables, 1 for finding antiderivatives, 1 for including a constant of integration AND using the initial condition to solve for it, 1 for an equation

Solution 4

involving G and t (i.e. solving for $G(t)$), 1 for the correct expression $G(t)$ and the correct value $G(3) = 54$. Note: max 2/5 if no constant of integration is included; 0/5 if variables are not separated.