Past-paper walk-through

Reasoning Using Slope Fields ▪ 7.4

eXercise

Questions in this set

- 2026 Q3
- 2024 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

A pie is taken from a hot oven and put on a table. The internal temperature of the pie at time t minutes can be modeled by the function H that satisfies the differential equation

$$\frac{dH}{dt} = -\frac{1}{15}(H - 20),$$

where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

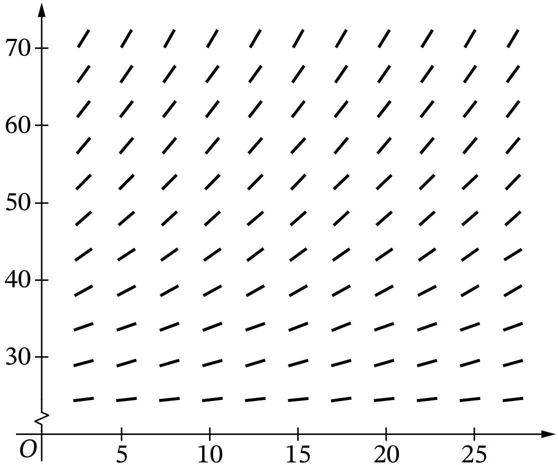
(a) Explain why the following could not be a slope field for the differential equation $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.

[Slope field graph: horizontal axis t from 0 to about 28 (gridlines at 5, 10, 15, 20, 25), vertical axis H from about 25 to 70 (gridlines at 30, 40, 50, 60, 70). Short line

Question 3

segments are drawn at each grid point. Near the top of the grid (around $H = 70$) the segments are steep, close to vertical, positive slope; moving down the grid the segments rotate, becoming shallow near $H = 30$, with negative slope by the bottom rows. At a given height H, the segments look the same regardless of t (slopes vary only with H, not with t, matching the autonomous equation) — but the segments are shown with a mix of orientations inconsistent with the required sign pattern of $\frac{dH}{dt} = -\frac{1}{15}(H - 20)$.]

Question 3



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where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(b) Find the slope of the line tangent to the graph of H at time $t = 0$. Show the work that leads to your answer.

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where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(c) It can be shown that $\frac{d^2H}{dt^2} = \frac{1}{225}(H - 20)$. The line tangent to the graph of H at time $t = 0$ is used to approximate $H(5)$, the internal temperature of the pie at time $t = 5$. Is this approximation an overestimate or an underestimate for the actual value of $H(5)$? Give a reason for your answer.

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where $H(t)$ is measured in degrees Celsius and $H(0) = 75$. For $t > 0$, it is known that $20 < H(t) < 75$.

(d) Use separation of variables to find an expression for $H(t)$, the particular solution to the given differential equation with initial condition $H(0) = 75$.

Question 3

2024 ▪ Q3

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = \dfrac{1}{2}(H-1)\cos\left(\dfrac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.

[Slope field graph; t-axis (horizontal) from 0 to 5 with gridlines at 1, 2, 3, 4, 5; y-axis (vertical) with gridlines at 5 and 10. A dot marks the point $(0, 4)$ on the y-axis. The slope field shows short line segments at grid points across the region $0 \leq t \leq 5$: for small y (near the t-axis) the segments are nearly horizontal (flat dashes); moving up toward $y = 5$–10, the segments tilt, transitioning from forward slashes ($/$, positive slope) on the left side of the grid (small t) through nearly vertical/horizontal ($-$ or $\sim$)

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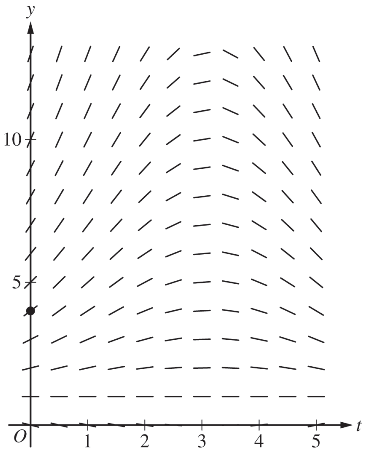
in the middle, to backslashes ($\backslash$, negative slope) on the right side (larger t), reflecting the $\cos(t/2)$ factor changing sign.]

Question 3



Question 3

2024 ▪ Q3

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(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

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2024 ▪ Q3

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\dfrac{dH}{dt} = \dfrac{1}{2}(H-1)\cos\left(\dfrac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon $(t=0)$. It is known that $H(0)=4$.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\dfrac{dH}{dt} = \dfrac{1}{2}(H-1)\cos\left(\dfrac{t}{2}\right)$ with initial condition $H(0)=4$.

Solution 3

(a)

Mark scheme

- This is a graphical sketching item: the solution curve y=H(t) must be drawn through the point (0,4), rising to a local maximum near t=$\pi(visuallyaroundH \approx 9 - 10onthegivenslopefield), thendecreasing, andmustextendtoatleastt = 4.5withnoobviousconflictswiththegivenslope-fieldsegments. Thesinglepointisearnedonlyforaha$ $drawncurvesatisfyingthesegraphicalrequirements; thereisnoseparatenumericoralgebraicanswer$

Solution 3

(b) Mark scheme

- Since H(t)>1 for 0<t<5, $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right) = 0 requires \cos(t/2) = 0, whichgivesthecriticalpointt t = \pi. For 0 < t < \pi, dH/dt > 0, and for \pi < t < 5, dH/dt < 0 (since H - 1 > 0 throughoutwhile \cos(t/2) changessignfrompositivetonegativeatt = \pi); therefore t = \pi isthelocationofarelativemaximumofH (equivalently, asecond - derivativetestatt = \pi givesd^2H/dt^2 < 0). Points: 1 forconsideringthesignofdH/dt (equivalentlydH/dt = 0, dH/dt > 0, dH/dt < 0, \cos(t/2) = 0, or \cos(t/2) > 0/ < 0), 1 foridentifyingt = \pi asthecriticalpoint (withorwithoutsupportingwork), 1 forthecorrectclassification' relativemaximu$ $\pi, oranequivalentsecond - derivativetest); thisthirdpointcannotbeearnedwithoutthefirst.$

Solution 3

(c) Mark scheme

- Separating variables: $\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right) dt. Integratingbothsides: \int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right) dt \;\Rightarrow\; \ln|H-1| = \sin\left(\frac{t}{2}\right) + C. UsingH(0) = 4 : \ln|4-1| = \sin(0) + C \Rightarrow C = \ln 3. SinceH(0) = 4 > 1, |H-1| = H-1, so\ln(H-1) = \sin(t/2) + \ln 3 \Rightarrow H-1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)} \Rightarrow H(t) = 1 + 3e^{\sin(t/2)}. Points(5 total, sequential): 1 forcorrectseparationofvariables, 1 foronecorrectantiderivative(eitherside), 1 forthesecondcorrect$ $4(eligibleonlyiftheseparationpointandatleastoneantiderivativepointwereearned), 1 forthecorrectf$ $formanswerH(t) = 1 + 3e^{\sin(t/2)}(eligibleonlyifallfirstfourpointswereearned).$