Past-paper walk-through

Verifying Solutions for Differential Equations ▪ 7.2

eXercise

Questions in this set

- 2015 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

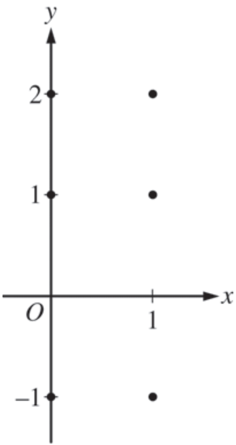
2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.

[Slope-field grid: x-axis marked at 1, y-axis marked at $-1, O, 1, 2$. Six dots (points where slope segments are to be drawn) are shown at coordinates $(1, 2)$, $(1, 1)$, and $(1, -1)$ — one column of three points at $x = 1$ for $y = 2, 1, -1$ — matching the "six points indicated" (the grid appears to repeat this column; all six indicated points lie at $x = 1$, with y-values $2, 1, -1$ shown, and the origin O marked at the axes intersection).]

Question 4



Question 4

2015 ▪ Q4

Consider the differential equation $\dfrac{dy}{dx} = 2x - y$.

(b) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

Question 4

2015 ▪ Q4

Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

Solution 4

(a)

Mark scheme

- The slope field uses $\frac{dy}{dx} = 2x - y$ evaluated at the six indicated points: at $x = 0$: $(0, 2) \to$ slope -2, $(0, 1) \to$ slope -1, $(0, -1) \to$ slope 1; at $x = 1$: $(1, 2) \to$ slope 0, $(1, 1) \to$ slope 1, $(1, -1) \to$ slope 3. Short line segments should be drawn with these slopes at each of the six points. Award 1 point for correct slope segments at the three points where $x = 0$, and 1 point for correct slope segments at the three points where $x = 1$.

Solution 4

(b)

Mark scheme

- $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - (2x - y) = 2 - 2x + y$. In Quadrant II, $x < 0$ and $y > 0$, so $2 - 2x + y > 0$. Therefore all solution curves are concave up in Quadrant II. Award 1 point for the correct expression for d^2y/dx^2 in terms of x and y, and 1 point for the correct concavity conclusion (concave up) with valid reasoning.

Solution 4

(c)

Mark scheme

- $\left.\frac{dy}{dx}\right|_{(x,y)=(2,3)} = 2(2) - 3 = 1 \neq 0.$ Therefore f has neither a relative minimum nor a relative maximum at $x = 2$. Award 1 point for evaluating dy/dx at $(2, 3)$, and 1 point for the correct conclusion (neither) with justification (the derivative is nonzero there).

Solution 4

(d) Mark scheme

- $y = mx + b \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(mx + b) = m$. Substituting into the differential equation: $2x - y = m \Rightarrow 2x - (mx + b) = m \Rightarrow (2 - m)x - (m + b) = 0$. This holds for all x only if $2 - m = 0$ and $m + b = 0$, giving $m = 2$ and $b = -2$. Award 1 point for $\frac{d}{dx}(mx + b) = m$, 1 point for setting up $2x - y = m$ (i.e., $2x - (mx + b) = m$), and 1 point for the correct answer $m = 2,\ b = -2$.