Past-paper walk-through

Exploring Accumulations of Change ▪ 6.1

eXercise

Questions in this set

- 2022 Q1
- 2018 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2022 ▪ Q1

From 5 A.M. to 10 A.M., the rate at which vehicles arrive at a certain toll plaza is given by $A(t) = 450\sqrt{\sin(0.62t)}$, where t is the number of hours after 5 A.M. and $A(t)$ is measured in vehicles per hour. Traffic is flowing smoothly at 5 A.M. with no vehicles waiting in line.

(a) Write, but do not evaluate, an integral expression that gives the total number of vehicles that arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(b) Find the average value of the rate, in vehicles per hour, at which vehicles arrive at the toll plaza from 6 A.M. ($t = 1$) to 10 A.M. ($t = 5$).

(c) Is the rate at which vehicles arrive at the toll plaza at 6 A.M. ($t = 1$) increasing or decreasing? Give a reason for your answer.

Question 1

(d) A line forms whenever $A(t) \geq 400$. The number of vehicles in line at time t, for $a \leq t \leq 4$, is given by

$$N(t) = \int_a^t (A(x) - 400)\, dx,$$

where a is the time when a line first begins to form. To the nearest whole number, find the greatest number of vehicles in line at the toll plaza in the time interval $a \leq t \leq 4$. Justify your answer.

Solution 1

(a)

Mark scheme

- The total number of vehicles arriving is given by the definite integral $\int_1^5 A(t)\,dt$ (do not evaluate). Scoring: 1 point for a definite integral with correct limits $t = 1$ to $t = 5$; a response using $|A(t)|$ or missing dt still earns the point since $A(t) \geq 0$ on this interval.

Solution 1

(b)

Mark scheme

- Average value $= \dfrac{1}{5-1}\int_1^5 A(t)\,dt = \dfrac{1}{4}(1502.147865) = 375.536966 \approx 375.537$ (or 375.536) vehicles per hour. 1 point for using the average value formula with correct limits $a = 1, b = 5$; 1 point for the correct numeric answer (must be accurate to three decimal places).

Solution 1

(c)

Mark scheme

- $A'(1) = 148.947272 > 0$, so the rate at which vehicles arrive at the toll plaza at $t = 1$ is increasing. 1 point for computing/considering $A'(1)$ (any value rounding to 148.947 to at least the presented decimal place); 1 point for the correct conclusion (increasing) with a reason that references $t = 1$.

Solution 1

(d)

Mark scheme

- Set $N'(t) = A(t) - 400 = 0 \Rightarrow A(t) = 400 \Rightarrow t = 1.469372$ $(= a)$ and $t = 3.597713$ $(= b)$. Evaluate $N(t) = \int_a^t (A(x) - 400)\, dx$: $N(a) = 0$, $N(b) = 71.254129$, $N(4) = 62.338346$. The greatest number of vehicles in line is 71 (to the nearest whole number, occurring at $t = b$). 4 points: 1 for setting $N'(t) = 0$, 1 for correctly identifying $t = a$ and $t = b$, 1 for the answer 71 (or 71.254...), 1 for justification (e.g. a candidates-test table comparing $N(a)$, $N(b)$, $N(4)$).

Question 1

2018 ▪ Q1

People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1-\frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

Question 1

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

Solution 1

(a)

Mark scheme

- $\int_0^{300} r(t)\,dt = 270$. According to the model, 270 people enter the line for the escalator during the time interval $0 \leq t \leq 300$. Points: 1 for setting up the integral of $r(t)$ over $[0, 300]$, 1 for the correct numeric answer (270).

Solution 1

(b)

Mark scheme

- $20 + \int_0^{300} (r(t) - 0.7)\, dt = 20 + \int_0^{300} r(t)\, dt - 0.7 \cdot 300 = 80$. According to the model, 80 people are in line at time $t = 300$. Points: 1 for considering the rate people leave the line (subtracting 0.7, the rate at which people get on the escalator) together with the initial 20 already in line, 1 for the correct answer (80).

Solution 1

(c)

Mark scheme

- Based on part (b), the number of people in line at $t = 300$ is 80. The first time t that there are no people in line is $300 + \dfrac{80}{0.7} = 414.286$ (or 414.285) seconds. 1 point for this answer (built on the part (b) result of 80 people in line, decreasing at the constant rate 0.7 people/second).

Solution 1

(d) Mark scheme

- The total number of people in line at time t, $0 \leq t \leq 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$. Setting $r(t) - 0.7 = 0$ gives the critical times $t_1 = 33.013298$ and $t_2 = 166.574719$ (but t_2 is out of the domain restriction used for the minimum check; the candidates evaluated are $t = 0, t_1, t_2, 300$). Evaluating the people-in-line function: at $t = 0$, 20 people; at $t_1 = 33.013$, 3.803 people; at t_2, 158.070 people; at $t = 300$, 80 people. The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line (rounded to the nearest whole number from 3.803). Points: 1 for considering $r(t) - 0.7 = 0$, 1 for identifying $t = 33.013$, 1 for the answer (4 people), 1 for justification (comparing candidate values, e.g. via a table, to confirm the minimum).