

Exercise walk-through

Integrating Using Integration by Parts ■ 6.11

eXercise

You must be able to

- apply $\int u \, dv = uv - \int v \, du$
- choose u and dv well (the **LIATE** guide)
- apply the method **twice** when needed

Method — The formula

Integration by parts reverses the product rule:

$$\int u \, dv = uv - \int v \, du.$$

Pick u (to differentiate) and dv (to integrate); then du and v follow.

Method — Choosing u with LIATE

Choose u as the **first** type in **LIATE** that appears: **L**ogs, **I**nverse trig, **A**lgebraic, **T**rig, **E**xponential. For $\int xe^x dx$: x is algebraic (A), e^x exponential (E), so $u = x$, $dv = e^x dx$.

Method — A worked example

$$\int x e^x dx: u = x, dv = e^x dx \Rightarrow du = dx, v = e^x:$$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C.$$

Method — Integrating a logarithm

$$\int \ln x \, dx: \text{ let } u = \ln x, \, dv = dx, \text{ so } du = \frac{1}{x} \, dx, \, v = x:$$

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C.$$

Practice 2.1 ■ 2 marks

For $\int x \cos x \, dx$, state a good choice of u and dv .

Solution 2.1

Worked steps

$u = x$ (algebraic), $dv = \cos x \, dx$ **M1** **A1**.

Practice 2.2 ■ 3 marks

Find $\int x \cos x \, dx$.

Solution 2.2

Worked steps

$$du = dx, v = \sin x; \int x \cos x dx = x \sin x - \int \sin x dx \quad \text{M1} = x \sin x + \cos x + C$$

M1 A1.

Practice 2.3 ■ 3 marks

Find $\int x e^{2x} dx$.

Solution 2.3

Worked steps

$$u = x, dv = e^{2x} dx \Rightarrow v = \frac{1}{2}e^{2x} \quad \text{M1}; \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x} dx \quad \text{M1} = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

In $\int x \sin x \, dx$, the best choice by LIATE is

A $u = \sin x, dv = x \, dx$

B $u = x, dv = \sin x \, dx$

C $u = x \sin x, dv = dx$

D $u = 1, dv = x \sin x \, dx$

Solution 3.1

Method: Choosing u with LIATE

A $u = \sin x, dv = x dx$

B $u = x, dv = \sin x dx$ 

C $u = x \sin x, dv = dx$

D $u = 1, dv = x \sin x dx$

Worked steps

B — x is algebraic (A), $\sin x$ is trig (T); A comes before T, so $u = x$.

Exam-style 3.2 ■ 3 marks

Find $\int \ln x \, dx$.

Solution 3.2

Worked steps

$$u = \ln x, dv = dx, v = x \quad \text{M1}; x \ln x - \int x \cdot \frac{1}{x} dx \quad \text{M1} = x \ln x - x + C \quad \text{A1}.$$

Exam-style 3.3 ■ 4 marks

Evaluate $\int_0^1 xe^x dx$.

Solution 3.3

Worked steps

$$u = x, dv = e^x dx; [xe^x]_0^1 - \int_0^1 e^x dx \quad \text{M1} = (e - 0) - [e^x]_0^1 \quad \text{M1} = e - (e - 1) \quad \text{M1} \\ = 1 \quad \text{A1}.$$