Past-paper walk-through

Using the Candidates Test to Determine Absolute (Global) Extrema ▪ 5.5

eXercise

Questions in this set

- 2026 Q4
- 2025 Q1
- 2025 Q4
- 2023 Q4
- 2022 Q3
- 2019 Q1
- 2019 Q3
- 2018 Q1
- 2017 Q3
- 2016 Q3

Questions in this set

- 2015 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2026 ▪ Q4

Let f be a twice-differentiable function on the closed interval $[-4, 4]$ with $f(2) = 3$. The graph of f', the derivative of f, is shown.

[Graph of f' in the xy-plane, x-axis from -4 to 4 (gridlines at each integer), y-axis from -1 to 5. The curve passes through labelled points: $(-4, -0.5)$, a local minimum at $(-3, -1)$, rising through the origin area, up to a local maximum at $(1, 3)$, back down through $(2, 1.5)$, continuing down to a local minimum at approximately $(3, 0)$ on the x-axis, then rising steeply up to $(4, 5)$. Caption: "Graph of f'".]

(a) For $x > 0$, the function g is defined by $g(x) = f(x) - \ln x$. Find $g'(2)$. Show the work that leads to your answer.

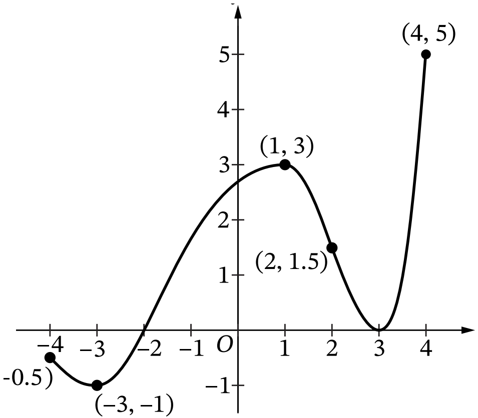
(b) Find all values of x on the open interval $0 < x < 3$ at which the graph of f has a point of inflection. Give a reason for your answer.

Question 4

(c) For $-4 \leq x \leq 4$, on what open intervals, if any, is the graph of f both increasing and concave down? Give a reason for your answer.

(d) For $-4 \leq x \leq 4$, find the value of x at which f has an absolute minimum and the value of x at which f has an absolute maximum. Give reasons for your answers.

Question 4



Graph of f'

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(a) Find the average number of acres affected by the invasive species from time $t = 0$ to time $t = 4$ weeks. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$. (Note: Your calculator should be in radian mode.)

(b) Find the time t when the instantaneous rate of change of C equals the average rate of change of C over the time interval $0 \leq t \leq 4$. Show the setup for your calculations.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6 \arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.

(Note: Your calculator should be in radian mode.)

(c) Assume that the invasive species continues to spread according to the given model for all times $t > 0$. Write a limit expression that describes the end behavior of the rate of change in the number of acres affected by the species. Evaluate this limit expression.

Question 1

2025 ▪ Q1

An invasive species of plant appears in a fruit grove at time $t = 0$ and begins to spread. The function C defined by $C(t) = 7.6\arctan(0.2t)$ models the number of acres in the fruit grove affected by the species t weeks after the species appears. It can be shown that $C'(t) = \dfrac{38}{25 + t^2}$.
(Note: Your calculator should be in radian mode.)

(d) At time $t = 4$ weeks after the invasive species appears in the fruit grove, measures are taken to counter the spread of the species. The function A, defined by $A(t) = C(t) - \displaystyle\int_4^t 0.1 \cdot \ln(x)\,dx$, models the number of acres affected by the species over the time interval $4 \le t \le 36$. At what time t, for $4 \le t \le 36$, does A attain its maximum value? Justify your answer.

Solution 1

(a)

Mark scheme

- Average value $= \dfrac{1}{4-0}\int_0^4 C(t)\,dt$. Since $\int_0^4 C(t)\,dt = 11.112896$, the average $= \dfrac{1}{4}(11.112896) = 2.778224 \approx 2.778$ acres. (1 pt: correct integral/average-value setup with division by 4; 1 pt: correct answer, accurate to 3 decimal places.)

Solution 1

(b)

Mark scheme

- Average rate of change of C on $[0, 4]$: $\dfrac{C(4) - C(0)}{4 - 0} = 1.282008$. Set $C'(t) = \dfrac{38}{25 + t^2}$ equal to this and solve: $\dfrac{38}{25 + t^2} = 1.282008 \Rightarrow t = 2.154298 \approx 2.154$. (1 pt: uses/presents the average rate of change; 1 pt: correct answer with supporting work.)

Solution 1

(c)

Mark scheme

- $\lim_{t\to\infty} C'(t) = \lim_{t\to\infty} \dfrac{38}{25+t^2} = 0$. (1 pt: correct limit expression $\lim_{t\to\infty} C'(t)$ or $\lim_{t\to\infty} C(t)$; 1 pt: correct value, 0.)

Solution 1

(d)

Mark scheme

- $A(t) = C(t) - \int_4^t 0.1 \ln x \, dx \Rightarrow A'(t) = C'(t) - 0.1 \ln t$. Setting $A'(t) = 0$: $C'(t) = 0.1 \ln t \Rightarrow t = 11.441700$. Candidates test on $[4, 36]$: $A(4) = 5.128031$, $A(11.4417) = 7.316978$, $A(36) = 1.743056$. The maximum occurs at $t = 11.442$ (or 11.441) weeks. (1 pt: considers $A'(t) = 0$; 1 pt: justification via global/candidates-test argument; 1 pt: correct answer with supporting work.)

Question 4

2025 ▪ Q4

The continuous function f is defined on the closed interval $-6 \leq x \leq 12$. The graph of f, consisting of two semicircles and one line segment, is shown in the figure. [Graph of f on the xy-plane, x-axis from -6 to 12 (gridlines at every integer), y-axis from -4 to 4 (gridlines at every integer). From $x = -6$ to $x = 0$: a semicircle below the x-axis, starting at the filled point $(-6, 0)$, dipping down to a minimum of about $(-3, -3)$, and returning up to $(0, 0)$. From $x = 0$ to $x = 6$: a semicircle above the x-axis, starting at $(0, 0)$, rising to a maximum of $(3, 3)$, and coming back down to $(6, 0)$. From $x = 6$ to $x = 12$: a straight line segment from $(6, 0)$ up to the filled point $(12, 3)$.]

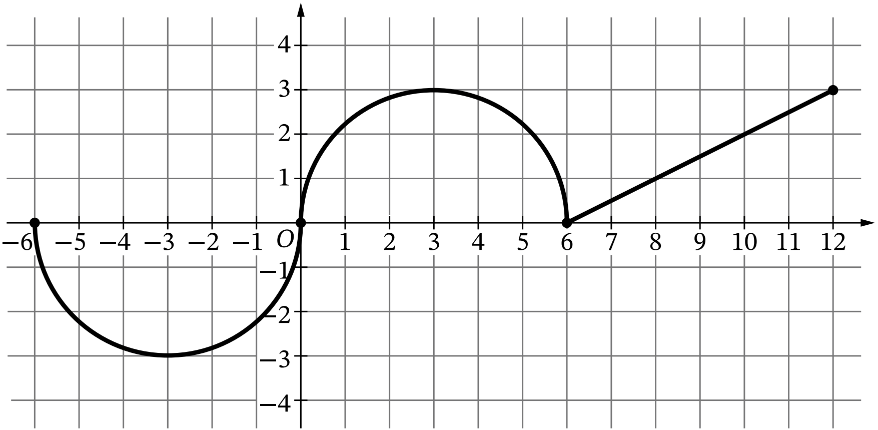
Let g be the function defined by $g(x) = \displaystyle\int_6^x f(t)\,dt$.

(a) Find $g'(8)$. Give a reason for your answer.

Question 4

(b) Find all values of x in the open interval $-6 < x < 12$ at which the graph of g has a point of inflection. Give a reason for your answer.
(c) Find $g(12)$ and $g(0)$. Label your answers.
(d) Find the value of x at which g attains an absolute minimum on the closed interval $-6 \leq x \leq 12$. Justify your answer.

Question 4



Graph of f

Solution 4

(a)

Mark scheme

- By the Fundamental Theorem of Calculus, $g'(x) = f(x)$. From the graph, $f(8) = 1$ (on the line segment from $(6, 0)$ to $(12, 3)$), so $g'(8) = f(8) = 1$. (1 pt: considers $g'(x) = f(x)$; 1 pt: correct answer, $g'(8) = 1$.)

Solution 4

(b)

Mark scheme

- g has a point of inflection where $g'' = f'$ changes sign, i.e. where $g' = f$ changes from decreasing to increasing or vice versa. From the graph, g has points of inflection at $x = -3$ and $x = 6$ (where f changes from decreasing to increasing) and at $x = 3$ (where f changes from increasing to decreasing). Values: $x = -3$, $x = 3$, $x = 6$. (1 pt: exactly these three x-values, no extras; 1 pt: reasoning explicitly tied to the graph of f, e.g. 'f changes from increasing to decreasing or decreasing to increasing there'.)

Solution 4

(c)

Mark scheme

- $g(12) = \int_6^{12} f(t)\,dt = \frac{1}{2}(6)(3) = 9$ (triangle area under the line segment). $g(0) = \int_6^0 f(x)\,dx = -\int_0^6 f(x)\,dx = -\frac{\pi}{2}(3)^2 = -\dfrac{9\pi}{2}$ (semicircle of radius 3). (1 pt: $g(12) = 9$, correctly labeled; 1 pt: $g(0) = -9\pi/2$, correctly labeled.)

Solution 4

(d)

Mark scheme

- g attains a minimum where $g'(x) = f(x) = 0$ or at an endpoint: $x = 0$ or $x = 6$ (from the graph). Candidates test: $g(-6) = 0$, $g(0) = -9\pi/2$, $g(6) = 0$, $g(12) = 9$. The absolute minimum value is $-9\pi/2$, occurring at $x = 0$. (1 pt: considers $g'(x) = 0$ (i.e. $f(x) = 0$); 1 pt: justification via global argument evaluating g at $x = -6, 0, 6, 12$; 1 pt: correct answer, $x = 0$.)

Question 4

2023 ▪ Q4

[Graph of f' on the xy-plane; x-axis from -2 to 8 with gridlines at each integer, y-axis with gridlines at $-2, -1, 1, 2$. The graph of f' consists of two line segments and a semicircle: a line segment from $(-2, 2)$ down to $(0, -2)$; a line segment from $(0, -2)$ up to $(4, 2)$; then a semicircle (below the segment, dipping down to touch the x-axis near $(6, 0)$) from $(4, 2)$ to $(8, 2)$.] Graph of f'

The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

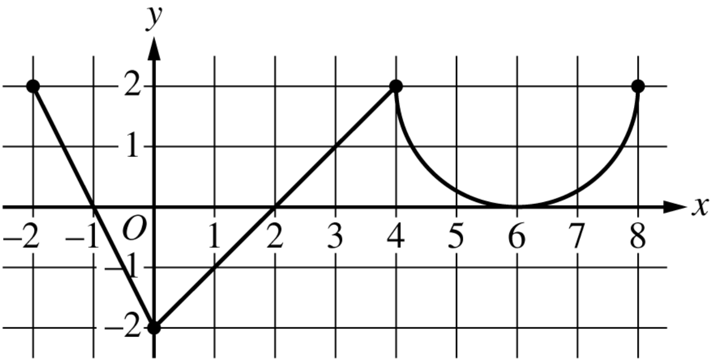
(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

Question 4

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

Question 4



Graph of f'

Solution 4

(a)

Mark scheme

- $f'(x) > 0$ on $(2, 6)$ and $f'(x) > 0$ on $(6, 8)$, so f' does not change sign at $x = 6$. Therefore f has neither a relative minimum nor a relative maximum at $x = 6$. (1 point for the correct answer 'neither' with the reason that f' does not change sign at $x = 6$; a response declaring $f'(x) > 0$ before and after $x = 6$ without stating this does not earn the point.)

Solution 4

(b)

Mark scheme

- The graph of f is concave down on $(-2, 0)$ and $(4, 6)$ because f' is decreasing on these intervals (from the graph of f', which consists of two line segments and a semicircle). (1 point for the correct intervals $(-2, 0)$ and $(4, 6)$, endpoints may be included or excluded; 1 point for the reason, correctly discussing the behavior of f' or the slopes of f' — this point requires the first point to be earned. A response with exactly one correct interval and a correct reason earns 1 of the 2 points.)

Solution 4

(c) Mark scheme

- Because f is differentiable at $x = 2$, f is continuous at $x = 2$, so $\lim_{x \to 2} f(x) = f(2) = 1$. Then $\lim_{x \to 2}(6f(x) - 3x) = 6(1) - 3(2) = 0$ and $\lim_{x \to 2}(x^2 - 5x + 6) = 0$. Since the limit is of indeterminate form $\frac{0}{0}$, L'Hopital's Rule applies: $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6} = \lim_{x \to 2} \frac{6f'(x) - 3}{2x - 5} = \frac{6 \cdot 0 - 3}{2 \cdot 2 - 5} = \frac{-3}{-1} = 3$. (1 point for presenting the two separate limits of numerator and denominator, both equal to 0 — a limit explicitly stated equal to $0/0$ does not earn this point; 1 point for applying L'Hopital's Rule, presenting at least one correct derivative in the limit of a ratio of derivatives; 1 point for the correct final answer 3 with supporting work.)

Solution 4

(d) Mark scheme

- $f'(x) = 0 \Rightarrow x = -1,\ x = 2,\ x = 6$. Since f is continuous on $[-2, 8]$, the candidates for the absolute minimum are $x = -2, -1, 2, 6, 8$. Values: $f(-2) = 3$, $f(-1) = 4$, $f(2) = 1$, $f(6) = 7 - \pi$, $f(8) = 11 - 2\pi$. The absolute minimum value of f is $f(2) = 1$. (1 point for stating $f'(x) = 0$ or equivalent — listing zeros of f' alone is not sufficient; 1 point for justification, i.e. evaluating f without error at all necessary critical points and both endpoints, where a value need not be presented if validly eliminated (e.g. $x = -1$ is a local max, $x = 6$ eliminated via part (a), or $f(8) > f(2)$ argued from $f'(x) \geq 0$ for $x > 2$) — not considering both endpoints forfeits this point; 1 point for the correct answer, stating the minimum VALUE is 1, not just the location $x = 2$.)

Question 3

2022 ▪ Q3

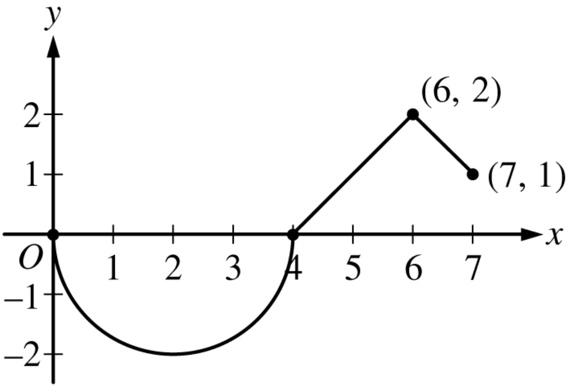
Let f be a differentiable function with $f(4) = 3$. On the interval $0 \leq x \leq 7$, the graph of f', the derivative of f, consists of a semicircle and two line segments, as shown in the figure above.
[Graph of f'; x-axis from 0 to 7 with tick marks at each integer, y-axis from -2 to 2 with tick marks at $-2, -1, 1, 2$. The graph starts at the origin $(0, 0)$, dips down as a semicircle below the x-axis reaching a minimum of $y = -2$ around $x = 2$, and returns to the x-axis at $(4, 0)$. From $(4, 0)$ a line segment rises to the labelled point $(6, 2)$, then a second line segment descends to the labelled point $(7, 1)$. Labeled "Graph of f'".]
(a) Find $f(0)$ and $f(5)$.
(b) Find the x-coordinates of all points of inflection of the graph of f for $0 < x < 7$. Justify your answer.

Question 3

(c) Let g be the function defined by $g(x) = f(x) - x$. On what intervals, if any, is g decreasing for $0 \leq x \leq 7$? Show the analysis that leads to your answer.

(d) For the function g defined in part (c), find the absolute minimum value on the interval $0 \leq x \leq 7$. Justify your answer.

Question 3



Graph of f'

Solution 3

(a)

Mark scheme

- $f(0) = f(4) + \int_4^0 f'(x)\,dx = 3 - \int_0^4 f'(x)\,dx = 3 - (-2\pi) = 3 + 2\pi$ (since $\int_0^4 f'(x)dx$ equals the negative of the semicircle area of radius 2, i.e. -2π). $f(5) = f(4) + \int_4^5 f'(x)\,dx = 3 + \frac{1}{2} = \frac{7}{2}$. 3 points: 1 for computing the area of either region (or an equivalent definite-integral expression), 1 for the correct $f(0) = 3 + 2\pi$, 1 for the correct $f(5) = 7/2$.

Solution 3

(b)

Mark scheme

- The graph of f has points of inflection at $x = 2$ and $x = 6$, because f' changes from decreasing to increasing at $x = 2$ (local min of f') and from increasing to decreasing at $x = 6$ (local max of f'). 2 points: 1 for stating both x-values, 1 for justification correctly tied to the behavior (slopes/local extrema) of the graph of f'.

Solution 3

(c)

Mark scheme

- $g(x) = f(x) - x \Rightarrow g'(x) = f'(x) - 1$. Setting $g'(x) \leq 0$: $f'(x) - 1 \leq 0 \Rightarrow f'(x) \leq 1$. From the graph, $f'(x) \leq 1$ on $0 \leq x \leq 5$, so g is decreasing on $0 \leq x \leq 5$. 2 points: 1 for $g'(x) = f'(x) - 1$, 1 for the correct interval $[0, 5]$ with supporting reasoning.

Solution 3

(d)

Mark scheme

- g is continuous with $g'(x) < 0$ for $0 < x < 5$ and $g'(x) > 0$ for $5 < x < 7$ (from part (c)), so the absolute minimum occurs at $x = 5$: $g(5) = f(5) - 5 = \frac{7}{2} - 5 = -\frac{3}{2}$. This is the absolute minimum value of g on $[0, 7]$. 2 points: 1 for identifying $x = 5$ as the only critical point via $g'(x) = 0$ (or equivalent sign-change reasoning), 1 for the correct value $-3/2$ with justification.

Question 1

2019 ▪ Q1

Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight $(t = 0)$.

(a) How many fish enter the lake over the 5-hour period from midnight $(t = 0)$ to 5 A.M. $(t = 5)$? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight $(t = 0)$ to 5 A.M. $(t = 5)$?

(c) At what time t, for $0 \leq t \leq 8$, is the greatest number of fish in the lake? Justify your answer.

Question 1

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

Solution 1

(a)

Mark scheme

- $\int_0^5 E(t)\,dt = 153.457690$. To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M. Points: 1 for the integral, 1 for the correct rounded answer (153).

Solution 1

(b)

Mark scheme

- $\dfrac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$. The average number of fish that leave the lake per hour over $[0, 5]$ is 6.059 fish per hour (accept 6.06). Points: 1 for the integral expression, 1 for the answer.

Solution 1

(c) Mark scheme

- The rate of change in the number of fish in the lake at time t is $E(t) - L(t)$. Setting $E(t) - L(t) = 0$ gives $t = 6.20356$. Since $E(t) - L(t) > 0$ for $0 \leq t < 6.20356$ and $E(t) - L(t) < 0$ for $6.20356 < t \leq 8$, the number of fish increases then decreases, so the greatest number of fish in the lake occurs at $t = 6.204$ (or 6.203). (Equivalent credit: let $A(t) = \int_0^t (E(s) - L(s))\,ds$ be the change in fish since midnight; $A'(t) = E(t) - L(t) = 0$ at $t = C = 6.20356$; comparing $A(0) = 0$, $A(C) = 135.01492$, $A(8) = 80.91998$ confirms the max is at $t = C$.) Points: 1 for setting $E(t) - L(t) = 0$, 1 for the answer $t = 6.204$, 1 for justification (sign analysis or candidates test).

Solution 1

(d)

Mark scheme

- $E'(5) - L'(5) = -10.7228 < 0$. Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish in the lake is decreasing at time $t = 5$. Points: 1 for considering both $E'(5)$ and $L'(5)$, 1 for the answer with explanation.

Question 3

2019 ▪ Q3

[Figure: Graph of f on a grid with x-axis from -2 to 5 and y-axis from -1 to 3. The graph consists of two line segments and a curve: a line segment from $(-2, 1)$ down to $(-1, -1)$; a line segment from $(-1, -1)$ up to $(2, 3)$; then a curve (quarter circle centered at $(5, 3)$) from $(2, 3)$ down to $(5, 0)$, passing through the point $(3, 3 - \sqrt{5})$.]

The continuous function f is defined on the closed interval $-6 \leq x \leq 5$. The figure above shows a portion of the graph of f, consisting of two line segments and a quarter of a circle centered at the point $(5, 3)$. It is known that the point $\left(3,\ 3 - \sqrt{5}\right)$ is on the graph of f.

(a) If $\displaystyle\int_{-6}^{5} f(x)\,dx = 7$, find the value of $\displaystyle\int_{-6}^{-2} f(x)\,dx$. Show the work that leads to your answer.

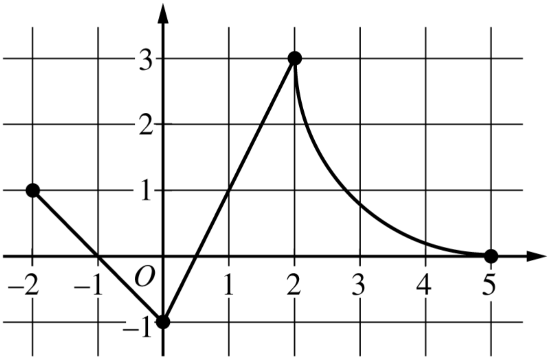
(b) Evaluate $\displaystyle\int_{3}^{5} \left(2f'(x) + 4\right) dx$.

Question 3

(c) The function g is given by $g(x) = \int_{-2}^{x} f(t)\,dt$. Find the absolute maximum value of g on the interval $-2 \le x \le 5$. Justify your answer.

(d) Find $\displaystyle\lim_{x\to 1} \frac{10^x - 3f'(x)}{f(x) - \arctan x}$.

Question 3



Graph of f

Solution 3

(a) Mark scheme

- $\int_{-6}^{5} f(x)\,dx = \int_{-6}^{-2} f(x)\,dx + \int_{-2}^{5} f(x)\,dx$. Given $\int_{-6}^{5} f(x)\,dx = 7$ and (from the graph of f) $\int_{-2}^{5} f(x)\,dx = 2 + \left(9 - \frac{9\pi}{4}\right)$, so
$7 = \int_{-6}^{-2} f(x)\,dx + 2 + \left(9 - \frac{9\pi}{4}\right) \Rightarrow \int_{-6}^{-2} f(x)\,dx = 7 - \left(11 - \frac{9\pi}{4}\right) = \frac{9\pi}{4} - 4.$
Points: 1 for splitting the integral as $\int_{-6}^{5} f = \int_{-6}^{-2} f + \int_{-2}^{5} f$, 1 for the value of $\int_{-2}^{5} f(x)\,dx$, 1 for the final answer.

Solution 3

(b)

Mark scheme

- $\int_3^5 (2f'(x)+4)\,dx = 2\int_3^5 f'(x)\,dx + \int_3^5 4\,dx = 2\big(f(5)-f(3)\big) + 4(5-3) = 2\big(0-(3-\sqrt{5})\big) + 8 = 2(-3+\sqrt{5}) + 8 = 2+2\sqrt{5}$ (using the Fundamental Theorem of Calculus with the given values $f(5)=0$, $f(3)=3-\sqrt{5}$). Points: 1 for applying the Fundamental Theorem of Calculus, 1 for the answer $2+2\sqrt{5}$.

Solution 3

(c) Mark scheme

- $g(x) = \int_{-2}^{x} f(t)\,dt$, so $g'(x) = f(x)$. Setting $g'(x) = f(x) = 0$ gives candidates $x = -1,\ x = \frac{1}{2},\ x = 5$ on $[-2, 5]$. Evaluating g at the candidates and the left endpoint: $g(-2) = 0$, $g(-1) = \frac{1}{2}$, $g(\frac{1}{2}) = -\frac{1}{4}$, $g(5) = 11 - \dfrac{9\pi}{4}$. On the interval $-2 \leq x \leq 5$, the absolute maximum value of g is $g(5) = 11 - \dfrac{9\pi}{4}$. Points: 1 for $g'(x) = f(x)$, 1 for identifying $x = -1$ as a candidate, 1 for the answer with justification (candidates/closed-interval test comparing g-values).

Solution 3

(d) Mark scheme

- $\lim_{x\to 1} \dfrac{10^x - 3f'(x)}{f(x) - \arctan x} = \dfrac{10^1 - 3f'(1)}{f(1) - \arctan 1} = \dfrac{10 - 3\cdot 2}{1 - \arctan 1} = \dfrac{4}{1-\pi/4}$ (using the given values $f(1) = 1$, $f'(1) = 2$; the limit can be evaluated directly by substitution since the denominator $f(1) - \arctan 1 \neq 0$). Points: 1 for the final answer $\dfrac{4}{1-\pi/4}$.

Question 1

2018 ▪ Q1

People enter a line for an escalator at a rate modeled by the function r given by

$$r(t) = \begin{cases} 44\left(\frac{t}{100}\right)^3\left(1 - \frac{t}{300}\right)^7 & \text{for } 0 \le t \le 300 \\ 0 & \text{for } t > 300, \end{cases}$$

where $r(t)$ is measured in people per second and t is measured in seconds. As people get on the escalator, they exit the line at a constant rate of 0.7 person per second. There are 20 people in line at time $t = 0$.

(a) How many people enter the line for the escalator during the time interval $0 \le t \le 300$?

(b) During the time interval $0 \le t \le 300$, there are always people in line for the escalator. How many people are in line at time $t = 300$?

Question 1

(c) For $t > 300$, what is the first time t that there are no people in line for the escalator?

(d) For $0 \leq t \leq 300$, at what time t is the number of people in line a minimum? To the nearest whole number, find the number of people in line at this time. Justify your answer.

Solution 1

(a)

Mark scheme

- $\int_0^{300} r(t)\,dt = 270$. According to the model, 270 people enter the line for the escalator during the time interval $0 \le t \le 300$. Points: 1 for setting up the integral of $r(t)$ over $[0, 300]$, 1 for the correct numeric answer (270).

Solution 1

(b)

Mark scheme

- $20 + \int_0^{300} (r(t) - 0.7)\, dt = 20 + \int_0^{300} r(t)\, dt - 0.7 \cdot 300 = 80$. According to the model, 80 people are in line at time $t = 300$. Points: 1 for considering the rate people leave the line (subtracting 0.7, the rate at which people get on the escalator) together with the initial 20 already in line, 1 for the correct answer (80).

Solution 1

(c)

Mark scheme

- Based on part (b), the number of people in line at $t = 300$ is 80. The first time t that there are no people in line is $300 + \dfrac{80}{0.7} = 414.286$ (or 414.285) seconds. 1 point for this answer (built on the part (b) result of 80 people in line, decreasing at the constant rate 0.7 people/second).

Solution 1

(d) Mark scheme

- The total number of people in line at time t, $0 \leq t \leq 300$, is modeled by $20 + \int_0^t r(x)\,dx - 0.7t$. Setting $r(t) - 0.7 = 0$ gives the critical times $t_1 = 33.013298$ and $t_2 = 166.574719$ (but t_2 is out of the domain restriction used for the minimum check; the candidates evaluated are $t = 0, t_1, t_2, 300$). Evaluating the people-in-line function: at $t = 0$, 20 people; at $t_1 = 33.013$, 3.803 people; at t_2, 158.070 people; at $t = 300$, 80 people. The number of people in line is a minimum at time $t = 33.013$ seconds, when there are 4 people in line (rounded to the nearest whole number from 3.803). Points: 1 for considering $r(t) - 0.7 = 0$, 1 for identifying $t = 33.013$, 1 for the answer (4 people), 1 for justification (comparing candidate values, e.g. via a table, to confirm the minimum).

Question 3

2017 ▪ Q3

[Graph of f': xy-plane with x-axis tick marks around -6 to past 3 and y-axis tick mark at 1, origin labeled O. The graph consists of a semicircle and three line segments. A line segment starts at the labeled point $(-6, 2)$ and decreases linearly to the x-axis at $x = -2$. From $x = -2$ to $x = 0$, a semicircle dips below the x-axis (minimum below the axis) and returns to the x-axis at $x = 0$. From $x = 0$ a line segment rises to the labeled point $(3, 2)$. From $(3, 2)$ a line segment decreases back down to the x-axis at $x = 5$. Caption: "Graph of f'".]

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

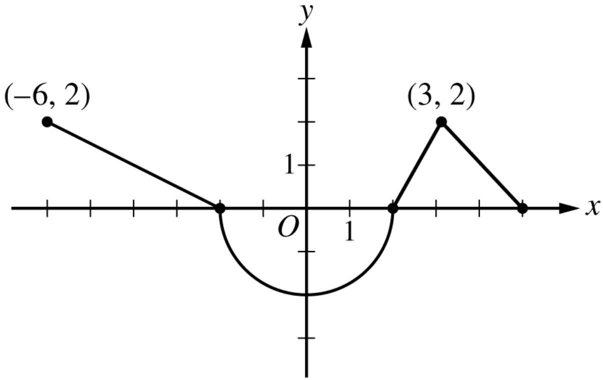
(a) Find the values of $f(-6)$ and $f(5)$.

(b) On what intervals is f increasing? Justify your answer.

Question 3

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

Question 3



Graph of f'

Solution 3

(a)

Mark scheme

- $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3.$
  $f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi.$ 1 pt for using the initial condition $f(-2) = 7$ with the Fundamental Theorem of Calculus, 1 pt for the correct $f(-6) = 3$, 1 pt for the correct $f(5) = 10 - 2\pi$.

Solution 3

(b)

Mark scheme

- $f'(x) > 0$ on the intervals $[-6, -2]$ and $(2, 5)$ (read from the given graph of f'). Therefore f is increasing on $[-6, -2]$ and $[2, 5]$. 2 pts for the correct intervals with justification tied to the sign of f'.

Solution 3

(c)

Mark scheme

- The absolute minimum occurs at a critical point where $f'(x) = 0$ or at an endpoint. $f'(x) = 0 \Rightarrow x = -2, x = 2$. Evaluating: $f(-6) = 3$, $f(-2) = 7$, $f(2) = 7 - 2\pi$, $f(5) = 10 - 2\pi$. The absolute minimum value is $f(2) = 7 - 2\pi$. 1 pt for considering $x = 2$ as a candidate critical point, 1 pt for the correct answer $7 - 2\pi$ with justification (comparing candidate values).

Solution 3

(d) Mark scheme

- $f'(-5) = \dfrac{2-0}{-6-(-2)} = -\dfrac{1}{2}$ (using the slope of f on that interval from the graph). For $f'(3)$: $\lim_{x\to 3^-} \dfrac{f(x)-f(3)}{x-3} = 2$ and $\lim_{x\to 3^+} \dfrac{f(x)-f(3)}{x-3} = -1$; since these one-sided limits differ, $f'(3)$ does not exist. 1 pt for $f'(-5) = -\frac{1}{2}$, 1 pt for correctly stating $f'(3)$ does not exist with this explanation (unequal one-sided difference-quotient limits).

Question 3

2016 ▪ Q3

The figure above shows the graph of the piecewise-linear function f. For $-4 \leq x \leq 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

[Graph of f: piecewise-linear function on axes with x from -4 to 12 (gridlines every 2) and y from -4 to 4. Key labeled points: $(-4, -4)$, then rising to $(0, 4)$, down to $(2, 0)$, up to $(4, 4)$, down to $(8, -4)$, up to $(10, 0)$ [labeled 10 on the x-axis], down to $(12, -4)$.]

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

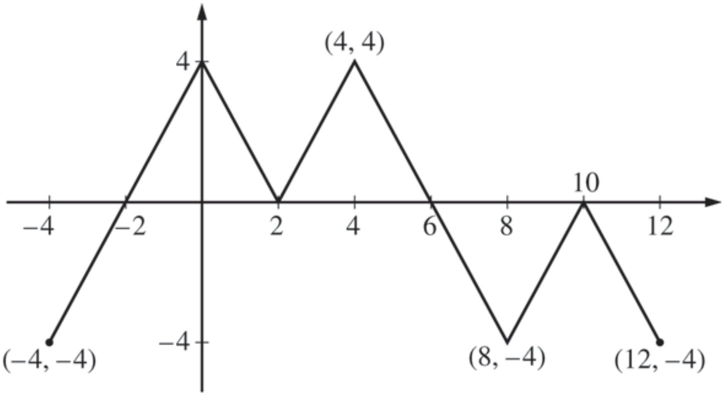
(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \leq x \leq 12$. Justify your answers.

Question 3

(d) For $-4 \leq x \leq 12$, find all intervals for which $g(x) \leq 0$.

Question 3



Graph of f

Solution 3

(a)

Mark scheme

- Since $g(x) = \int_2^x f(t)\,dt$, the Fundamental Theorem of Calculus gives $g'(x) = f(x)$ (this key relationship is worth 1 point, creditable in any part of the question). On $8 \leq x \leq 12$ the graph shows $f(x) \leq 0$, touching 0 only at $x = 10$, so $g'(x) = f(x)$ does not change sign at $x = 10$. Therefore g has NEITHER a relative minimum NOR a relative maximum at $x = 10$. Award 1 point for establishing $g'(x) = f(x)$, and 1 point for the correct answer with justification.

Solution 3

(b)

Mark scheme

- $g'(x) = f(x)$ is increasing for $2 \leq x \leq 4$ and decreasing for $4 \leq x \leq 8$ (read from the graph of f), so g is concave up then concave down at $x = 4$. So YES, the graph of g has a point of inflection at $x = 4$. Award 1 point for the answer with justification.

Solution 3

(c) Mark scheme

- $g'(x) = f(x)$ changes sign only at $x = -2$ and $x = 6$, so these are candidates for extrema; the endpoints $x = -4$ and $x = 12$ must also be considered. Evaluating g (the signed area under f from 2 to x, from the graph) at each candidate gives the table $g(-4) = -4$, $g(-2) = -8$, $g(6) = 8$, $g(12) = -4$. On $-4 \leq x \leq 12$, the absolute minimum value is $g(-2) = -8$ and the absolute maximum value is $g(6) = 8$. Award 1 point for considering $x = -2$ and $x = 6$ as candidates, 1 point for considering the endpoints $x = -4$ and $x = 12$ as candidates, and 2 points for the two correct answers with justification (citing the candidate-value table).

Solution 3

(d)

Mark scheme

- $g(x) \leq 0$ for $-4 \leq x \leq 2$ and for $10 \leq x \leq 12$, matching the sign of g determined from the candidate-value table (g is ≤ 0 from the left endpoint through $x = 2$, where $g(2) = 0$ by definition, and again from $x = 10$, where g returns to 0, through the right endpoint $x = 12$). Award 2 points for correctly identifying both intervals.

Question 1

2015 ▪ Q1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.

Question 1

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

Solution 1

(a)

Mark scheme

- The total rainwater inflow is $\int_0^8 R(t)\,dt = 76.570$ cubic feet (accept 76.569–76.571). Award 1 point for setting up the correct integrand $R(t)$, and 1 point for the correct numeric answer.

Solution 1

(b)

Mark scheme

- $R(3) - D(3) = -0.313632 < 0$. Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours. Award 1 point for considering/evaluating both $R(3)$ and $D(3)$, and 1 point for the correct answer (decreasing) with correct reasoning ($R(3) < D(3)$).

Solution 1

(c)

Mark scheme

- The amount of water in the pipe at time t is $30 + \int_0^t [R(x) - D(x)]\, dx$. Setting $R(t) - D(t) = 0$ gives $t = 0$ or $t = 3.271658$. Evaluating the amount of water at the critical point and endpoints: $t = 0 \Rightarrow 30$; $t = 3.271658 \Rightarrow 27.964561$; $t = 8 \Rightarrow 48.543686$. The minimum occurs at $t = 3.272$ (or 3.271) hours. Award 1 point for considering $R(t) - D(t) = 0$, 1 point for the correct answer $t \approx 3.271$ or 3.272, and 1 point for justification (comparing values at the candidate and endpoints).

Solution 1

(d)

Mark scheme

- $30 + \int_0^w [R(t) - D(t)]\, dt = 50$. Award 1 point for the correct integral expression $\int_0^w [R(t) - D(t)]\, dt$, and 1 point for setting the full equation equal to 50 (the equation is not required to be solved).