Past-paper walk-through

Introduction to Related Rates ▪ 4.4

eXercise

Questions in this set

- 2022 Q4
- 2018 Q4

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2022 ▪ Q4

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

An ice sculpture melts in such a way that it can be modeled as a cone that maintains a conical shape as it decreases in size. The radius of the base of the cone is given by a twice-differentiable function r, where $r(t)$ is measured in centimeters and t is measured in days. The table above gives selected values of $r'(t)$, the rate of change of the radius, over the time interval $0 \leq t \leq 12$.

(a) Approximate $r''(8.5)$ using the average rate of change of r' over the interval $7 \leq t \leq 10$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time t, $0 \leq t \leq 3$, for which $r'(t) = -6$? Justify your answer.

Question 4

(c) Use a right Riemann sum with the four subintervals indicated in the table to approximate the value of

$$\int_0^{12} r'(t)\,dt.$$

(d) The height of the cone decreases at a rate of 2 centimeters per day. At time $t = 3$ days, the radius is 100 centimeters and the height is 50 centimeters. Find the rate of change of the volume of the cone with respect to time, in cubic centimeters per day, at time $t = 3$ days. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

Question 4

t (days)	0	3	7	10	12
$r'(t)$ (centimeters per day)	-6.1	-5.0	-4.4	-3.8	-3.5

Solution 4

(a)

Mark scheme

- $r''(8.5) \approx \dfrac{r'(10) - r'(7)}{10 - 7} = \dfrac{-3.8 - (-4.4)}{3} = \dfrac{0.6}{3} = 0.2$ centimeters per day per day. 2 points: 1 for the correct difference-quotient setup and value (supporting work must include at least a difference and a quotient), 1 for correct units (cm/day^2).

Solution 4

(b)

Mark scheme

- $r'(0) = -6.1 < -6 < -5.0 = r'(3)$. Since r is twice differentiable, r' is differentiable and therefore continuous on $[0, 3]$. By the Intermediate Value Theorem, there is a time t, $0 < t < 3$, such that $r'(t) = -6$. 2 points: 1 for establishing that -6 lies between $r'(0)$ and $r'(3)$, 1 for the continuity statement plus IVT conclusion that such a t exists.

Solution 4

(c)

Mark scheme

- Right Riemann sum with subintervals $[0,3],[3,7],[7,10],[10,12]$: $\int_0^{12} r'(t)\,dt \approx 3r'(3)+4r'(7)+3r'(10)+2r'(12) = 3(-5.0)+4(-4.4)+3(-3.8)+2(-3.5) = -15-17.6-11.4-7 = -51$. 2 points: 1 for the correct form of the right Riemann sum (at least 7 of the 8 factors correct), 1 for the correct value -51 (units not required).

Solution 4

(d) Mark scheme

- $V = \frac{1}{3}\pi r^2 h \Rightarrow \dfrac{dV}{dt} = \frac{2}{3}\pi rh\dfrac{dr}{dt} + \frac{1}{3}\pi r^2\dfrac{dh}{dt}$ (product rule on r^2h, chain rule for each differential). At $t = 3$: $r = 100$, $h = 50$, $\dfrac{dr}{dt} = r'(3) = -5$, $\dfrac{dh}{dt} = -2$ (height decreasing 2 cm/day). $\left.\dfrac{dV}{dt}\right|_{t=3} = \frac{2}{3}\pi(100)(50)(-5) + \frac{1}{3}\pi(100)^2(-2) = -\dfrac{70{,}000\pi}{3}$ cubic centimeters per day. 3 points: 1 for the product-rule term, 1 for the chain-rule differentials, 1 for the correct final answer (both prior points required to earn this one).

Question 4

2018 ▪ Q4

t **(years)**	**2**	**3**	**5**	**7**	**10**
$H(t)$ (meters)	1.5	2	6	11	15

The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

Question 4

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

Question 4

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15