Past-paper walk-through

Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist ▪ 2.4

eXercise

Questions in this set

- 2017 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2017 ▪ Q3

[Graph of f': xy-plane with x-axis tick marks around -6 to past 3 and y-axis tick mark at 1, origin labeled O. The graph consists of a semicircle and three line segments. A line segment starts at the labeled point $(-6, 2)$ and decreases linearly to the x-axis at $x = -2$. From $x = -2$ to $x = 0$, a semicircle dips below the x-axis (minimum below the axis) and returns to the x-axis at $x = 0$. From $x = 0$ a line segment rises to the labeled point $(3, 2)$. From $(3, 2)$ a line segment decreases back down to the x-axis at $x = 5$. Caption: "Graph of f'".]

The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f', the derivative of f, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find the values of $f(-6)$ and $f(5)$.

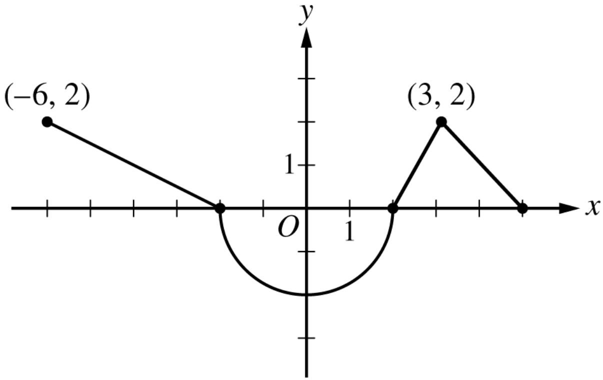
(b) On what intervals is f increasing? Justify your answer.

Question 3

(c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.

(d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.

Question 3



Graph of f'

Solution 3

(a)

Mark scheme

- $f(-6) = f(-2) + \int_{-2}^{-6} f'(x)\,dx = 7 - \int_{-6}^{-2} f'(x)\,dx = 7 - 4 = 3.$
  $f(5) = f(-2) + \int_{-2}^{5} f'(x)\,dx = 7 - 2\pi + 3 = 10 - 2\pi.$ 1 pt for using the initial condition $f(-2) = 7$ with the Fundamental Theorem of Calculus, 1 pt for the correct $f(-6) = 3$, 1 pt for the correct $f(5) = 10 - 2\pi$.

Solution 3

(b)

Mark scheme

- $f'(x) > 0$ on the intervals $[-6, -2]$ and $(2, 5)$ (read from the given graph of f'). Therefore f is increasing on $[-6, -2]$ and $[2, 5]$. 2 pts for the correct intervals with justification tied to the sign of f'.

Solution 3

(c)

Mark scheme

- The absolute minimum occurs at a critical point where $f'(x) = 0$ or at an endpoint. $f'(x) = 0 \Rightarrow x = -2, x = 2$. Evaluating: $f(-6) = 3$, $f(-2) = 7$, $f(2) = 7 - 2\pi$, $f(5) = 10 - 2\pi$. The absolute minimum value is $f(2) = 7 - 2\pi$. 1 pt for considering $x = 2$ as a candidate critical point, 1 pt for the correct answer $7 - 2\pi$ with justification (comparing candidate values).

Solution 3

(d) Mark scheme

- $f'(-5) = \dfrac{2-0}{-6-(-2)} = -\dfrac{1}{2}$ (using the slope of f on that interval from the graph). For $f'(3)$: $\displaystyle\lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3} = 2$ and $\displaystyle\lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3} = -1$; since these one-sided limits differ, $f'(3)$ does not exist. 1 pt for $f'(-5) = -\frac{1}{2}$, 1 pt for correctly stating $f'(3)$ does not exist with this explanation (unequal one-sided difference-quotient limits).