Past-paper walk-through

Working with Geometric Series ▪ 10.2

eXercise

Questions in this set

- 2026 Q6
- 2025 Q6
- 2022 Q6
- 2014 Q6

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 6

2026 ▪ Q6

The Maclaurin series for a function g is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$$

and converges to $g(x)$ for $|x| < 5$.

(a) For $x > 0$, the Maclaurin series for g is an alternating geometric series. Find $g(3)$. Show the work that leads to your answer.

Question 6

2026 ▪ Q6

The Maclaurin series for a function g is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$$

and converges to $g(x)$ for $|x| < 5$.

(b) The function f is defined by $f(x) = g'(x)$. Write the first four nonzero terms of the Maclaurin series for f.

Question 6

2026 ▪ Q6

The Maclaurin series for a function g is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$$

and converges to $g(x)$ for $|x| < 5$.

(c) The second-degree Taylor polynomial for f about $x = 0$ is used to approximate $f\left(\frac{5}{2}\right)$ as $-\frac{3}{10}$, where f is the function defined in part B. Justify that $\left|f\left(\frac{5}{2}\right) - \left(-\frac{3}{10}\right)\right| \leq \frac{1}{5}$.

Question 6

2026 ▪ Q6

The Maclaurin series for a function g is given by

$$\sum_{n=0}^{\infty} 2\left(\frac{-x}{5}\right)^n = 2 - \frac{2}{5}x + \frac{2}{25}x^2 - \frac{2}{125}x^3 + \frac{2}{625}x^4 - \cdots + 2\left(\frac{-x}{5}\right)^n + \cdots$$

and converges to $g(x)$ for $|x| < 5$.

(d)(i) The function h is defined by $h(x) = 25g(x) - 2e^x$. Write the first three nonzero terms of the Maclaurin series for e^x.

(d)(ii) Write the first three nonzero terms of the Maclaurin series for h.

Question 6

2025 ▪ Q6

The Taylor series for a function f about $x = 4$ is given by

$$\sum_{n=1}^{\infty} \frac{(x-4)^{n+1}}{(n+1)3^n} = \frac{(x-4)^2}{2 \cdot 3} + \frac{(x-4)^3}{3 \cdot 3^2} + \frac{(x-4)^4}{4 \cdot 3^3} + \cdots + \frac{(x-4)^{n+1}}{(n+1)3^n} + \cdots$$

and converges to $f(x)$ on its interval of convergence.

(a) Using the ratio test, find the interval of convergence of the Taylor series for f about $x = 4$. Justify your answer.

(b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 4$.

(c) The Taylor series for f' described in part B is a geometric series. For all x in the interval of convergence of the Taylor series for f', show that $f'(x) = \dfrac{x-4}{7-x}$.

Question 6

(d) It is known that the radius of convergence of the Taylor series for f about $x = 4$ is the same as the radius of convergence of the Taylor series for f' about $x = 4$. Does the Taylor series for f' described in part B converge to $f'(x) = \dfrac{x-4}{7-x}$ at $x = 8$? Give a reason for your answer.

Solution 6

(a) Mark scheme

- Ratio test: $\left|\dfrac{(x-4)^{n+2}}{(n+2)3^{n+1}} \cdot \dfrac{(n+1)3^n}{(x-4)^{n+1}}\right| = \left|\dfrac{(x-4)(n+1)}{3(n+2)}\right|$. Limit as $n \to \infty$: $\left|\dfrac{x-4}{3}\right|$. This is < 1 when $|x-4| < 3$, i.e. interior $1 < x < 7$. At $x = 1$: series is $\sum \dfrac{(-1)^{n+1} \cdot 3}{n+1}$, converges by the alternating series test. At $x = 7$: series is $\sum \dfrac{3}{n+1}$, diverges by limit comparison to the harmonic series. Interval of convergence: $1 \leq x < 7$. (1 pt: sets up the ratio; 1 pt: correct limit of the ratio, $|x-4|/3$; 1 pt: correct interior of interval, $1 < x < 7$; 1 pt: considers both

Solution 6

endpoints $x = 1$ and $x = 7$; 1 pt: correct analysis at both endpoints and correct final interval $1 \leq x < 7$.)

Solution 6

(b)

Mark scheme

- Differentiating the given series term by term:
  $f'(x) = \frac{x-4}{3} + \frac{(x-4)^2}{9} + \frac{(x-4)^3}{27} + \cdots + \frac{(x-4)^n}{3^n} + \cdots$. (1 pt: correct first three nonzero terms; 1 pt: correct general term, $(x-4)^n/3^n$.)

Solution 6

(c) Mark scheme

- The series for f is geometric with first term $\frac{x-4}{3}$ and common ratio $\frac{x-4}{3}$, so $f(x) = \frac{\frac{x-4}{3}}{1-\frac{x-4}{3}} = \frac{x-4}{3-(x-4)} = \frac{x-4}{7-x}$. (1 pt: verification — identifies first term/ratio of the geometric series and simplifies to the given closed form.)

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(d)

Mark scheme

- The interior of the interval of convergence found in part A is $1 < x < 7$, and (given) the Taylor series for f' has the same radius of convergence. Since $x = 8$ is outside $1 < x < 7$, the Taylor series for f' does NOT converge to $f'(x) = (x-4)/(7-x)$ at $x = 8$. (1 pt: correct 'no' answer with a reason based on $x = 8$ lying outside the interval of convergence.)

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2022 ▪ Q6

The function f is defined by the power series $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + \frac{(-1)^n x^{2n+1}}{2n+1} + \cdots$ for all real numbers x for which the series converges.

(a) Using the ratio test, find the interval of convergence of the power series for f. Justify your answer.

(b) Show that $\left| f\left(\frac{1}{2}\right) - \frac{1}{2} \right| < \frac{1}{10}$. Justify your answer.

(c) Write the first four nonzero terms and the general term for an infinite series that represents $f'(x)$.

(d) Use the result from part (c) to find the value of $f'\left(\frac{1}{6}\right)$.

Solution 6

(a) Mark scheme

- Ratio test: $\lim_{n\to\infty}\left|\dfrac{(-1)^{n+1}x^{2n+3}/(2n+3)}{(-1)^n x^{2n+1}/(2n+1)}\right| = \lim_{n\to\infty}\left|x^2\dfrac{2n+1}{2n+3}\right| = |x^2| = x^2$. Need $x^2 < 1 \Rightarrow -1 < x < 1$ (interior of interval). At $x = -1$: series is $-1 + \frac{1}{3} - \frac{1}{5} + \cdots$, alternating with terms decreasing in absolute value to 0, converges by the Alternating Series Test. At $x = 1$: series is $1 - \frac{1}{3} + \frac{1}{5} - \cdots$, also alternating with terms decreasing to 0, converges by AST. So the interval of convergence is $-1 \leq x \leq 1$. 4 points: 1 for setting up the ratio, 1 for identifying the interior $-1 < x < 1$ from the limit, 1 for considering both endpoints ($x = \pm 1$), 1 for correct analysis at both endpoints and stating the full interval $-1 \leq x \leq 1$.

Solution 6

(b) Mark scheme

- $f(1/2)$ is an alternating series with first term $\frac{1}{2}$ and terms decreasing in absolute value to 0. By the alternating series error bound, $\left|f(\frac{1}{2}) - \frac{1}{2}\right| < |\text{second term}| = \left|\dfrac{(-1)^1(1/2)^3}{3}\right| = \dfrac{1}{24} < \dfrac{1}{10}$. 2 points: 1 for correctly using $x = \frac{1}{2}$ in the second term of the series, 1 for stating the series is alternating with terms decreasing to 0 and presenting the inequality Error $< \frac{1}{24} < \frac{1}{10}$ (stating Error $= 1/24$ alone does not earn this point).

Solution 6

(c)

Mark scheme

- $f'(x) = 1 - x^2 + x^4 - x^6 + \cdots + (-1)^n x^{2n} + \cdots$ (term-by-term derivative of the series for f). 2 points: 1 for presenting the first four nonzero terms $(1, -x^2, x^4, -x^6)$, 1 for the correct general term $(-1)^n x^{2n}$.

Solution 6

(d) Mark scheme

- $f(1/6) = 1 - (1/6)^2 + (1/6)^4 - (1/6)^6 + \cdots$ is a geometric series with first term $a = 1$ and common ratio $r = -1/36$.
  $f(1/6) = \dfrac{a}{1-r} = \dfrac{1}{1-(-1/36)} = \dfrac{1}{37/36} = \dfrac{36}{37}$. 1 point for the correct answer $36/37$, contingent on correctly identifying the series from part (c) as geometric.

Question 6

2014 ▪ Q6

The Taylor series for a function f about $x = 1$ is given by $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{n} (x-1)^n$ and converges to $f(x)$ for $|x-1| < R$, where R is the radius of convergence of the Taylor series.

(a) Find the value of R.

(b) Find the first three nonzero terms and the general term of the Taylor series for f', the derivative of f, about $x = 1$.

(c) The Taylor series for f' about $x = 1$, found in part (b), is a geometric series. Find the function f' to which the series converges for $|x-1| < R$. Use this function to determine f for $|x-1| < R$.