Exercise walk-through

Finding Taylor or Maclaurin Series for a Function ▪ 10.14

eXercise

You must be able to

- recall the key **Maclaurin series** for e^x, $\sin x$, $\cos x$, $\dfrac{1}{1-x}$
- build new series by **substitution** into a known one
- write a series in summation form

Method — The must-know Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

Method — Building by substitution

For e^{x^2}, substitute x^2 into the e^x series:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}.$$

Method — Another substitution

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n.$$

Method — Writing out terms

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$. Being able to write the first few terms **and** the general summation form is expected.

Practice 2.1 ▪ 1 mark

Write the Maclaurin series for e^x (summation form).

Solution 2.1

Method: Writing out terms

Worked steps

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \boxed{\textbf{B1}}.$$

Practice 2.2 ▪ 2 marks

Write the first three nonzero terms of $\cos x$.

Solution 2.2

Method: Writing out terms

Worked steps

$$1 - \frac{x^2}{2} + \frac{x^4}{24} \quad \text{M1}\ \text{A1}.$$

Practice 2.3 ▪ 2 marks

Use substitution to write the Maclaurin series for e^{-x}.

Solution 2.3

Worked steps

$$\sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} \quad \text{M1} \ \text{A1}.$$

Exam-style 3.1 ▪ 1 mark

The Maclaurin series for $\dfrac{1}{1-x}$ is

A $\sum(-1)^n x^n$

B $\sum x^n$

C $\sum \dfrac{x^n}{n!}$

D $\sum nx^n$

Solution 3.1

Method: The must-know Maclaurin series

A $\sum(-1)^n x^n$

B $\sum x^n$ ✓

C $\sum \frac{x^n}{n!}$

D $\sum nx^n$

Worked steps

B — $\dfrac{1}{1-x} = \sum x^n$.

Exam-style 3.2 ▪ 3 marks

Find the Maclaurin series for $\dfrac{1}{1+x^2}$ (substitute into the geometric series).

Solution 3.2

Worked steps

$$\frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n} \quad \text{M1 M1 A1}.$$

Exam-style 3.3 ▪ 3 marks

Find the first three nonzero terms of the Maclaurin series for $x \sin x$.

Solution 3.3

Worked steps

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$; multiply by x: $x^2 - \frac{x^4}{6} + \frac{x^6}{120} - \cdots$ **M1** **M1** **A1**.