

9. Parametric Equations, Polar Coordinates, and Vector-Valued Functions

Starter Quiz ·

AP Calculus BC

Name: _____ Score: _____

1. For a parametric curve, $\frac{dy}{dx} =$

☐ $\frac{dy}{dt} \cdot \frac{dx}{dt}$

☐ $\frac{d}{dt} \left[\frac{y}{x} \right]$

☐ $\frac{dx/dt}{dy/dt}$

☐ $\frac{dy/dt}{dx/dt}$

2. The parametric second derivative $\frac{d^2y}{dx^2}$ equals...

☐ $\frac{d^2y}{dt^2}$

☐ $\frac{dy/dt}{dx/dt}$

☐ $\frac{d^2y/dt^2}{d^2x/dt^2}$

☐ $\frac{\frac{d}{dt}(dy/dx)}{dx/dt}$

3. The arc length of a parametric curve is $\int_a^b (\quad) dt$ where the integrand is...

☐ $\sqrt{(dx/dt)^2 - (dy/dt)^2}$

☐ $(dx/dt) + (dy/dt)$

☐ $\sqrt{(dx/dt)^2 + (dy/dt)^2}$

☐ $\sqrt{1 + (dy/dx)^2}$

4. To differentiate $\mathbf{r}(t) = \langle x(t), y(t) \rangle$, you...

☐ add the components then differentiate

☐ multiply the components

☐ take the magnitude first

☐ differentiate each component separately

5. To integrate $\langle f(t), g(t) \rangle$, you...

☐ take the magnitude

☐ differentiate instead

☐ multiply the components

☐ integrate each component separately

6. For $\mathbf{v}(t) = \langle 2t, 3 \rangle$, find the speed at $t = 2$.

7. Convert $(2, \frac{\pi}{3})$: find the x -coordinate.

8. Find the area enclosed by $r = 2$: $\frac{1}{2} \int_0^{2\pi} 4 \, d\theta$ (as a multiple of π , enter the number).

9. Area inside $r = 2$, outside $r = 1$: $\frac{1}{2} \int_0^{2\pi} (4 - 1) \, d\theta$ (as a multiple of π).

10. For $x = t^2$, $y = t^3$, $\frac{dy}{dx} = \frac{3t}{2}$. Find the slope at $t = 2$.

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Answers ·

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1. $\frac{dy/dt}{dx/dt}$

Top over bottom: $\frac{dy/dt}{dx/dt}$.

2. $\frac{\frac{d}{dt}(dy/dx)}{dx/dt}$

Differentiate $\frac{dy}{dx}$ in t , then divide by $\frac{dx}{dt}$.

3. $\sqrt{(dx/dt)^2 + (dy/dt)^2}$

Both derivatives squared, added, under a root.

4. **differentiate each component separately**

$$\mathbf{r}' = \langle x', y' \rangle.$$

5. **integrate each component separately**

Integrate componentwise, plus a constant vector.

6. **5**

$$\sqrt{4^2 + 3^2} = 5.$$

7. **1**

$$2 \cos \frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1.$$

8. **4**

$$\frac{1}{2} \cdot 4 \cdot 2\pi = 4\pi.$$

9. **3**

$$\frac{1}{2} \cdot 3 \cdot 2\pi = 3\pi.$$

10. **3**

$$\frac{3(2)}{2} = 3.$$