

3. Differentiation: Composite, Implicit, and Inverse Functions

Starter Quiz ·

AP Calculus BC

Name: _____ Score: _____

1. The Chain Rule gives $\frac{d}{dx}[f(g(x))] =$

☐ $f'(g(x))$

☐ $f'(x) \cdot g'(x)$

☐ $f(g'(x))$

☐ $f'(g(x)) \cdot g'(x)$

2. Differentiating with respect to x , $\frac{d}{dx}[y^2] =$

☐ $2x$

☐ $2y$

☐ $2y \frac{dy}{dx}$

☐ 2

3. The derivative of an inverse is $(f^{-1})'(x) =$

☐ $f'(x)$

☐ $\frac{1}{f'(x)}$

☐ $-f'(x)$

☐ $\frac{1}{f'(f^{-1}(x))}$

4. What is $\frac{d}{dx}[\arcsin x]$?

☐ $\cos x$

☐ $\frac{1}{\sqrt{1-x^2}}$

☐ $-\frac{1}{\sqrt{1-x^2}}$

☐ $\frac{1}{1+x^2}$

5. Which rule does $y = x^2 \sin x$ need at the top level?

☐ chain rule only

☐ product rule

☐ quotient rule

☐ power rule only

6. For $f(x) = x^3 - 2x^2 + 5x$, $f'(x) = 6x - 4$. Find $f'(2)$.

7. For $y = (2x + 1)^3$, find y' at $x = 0$. (Chain rule: $3(2x + 1)^2 \cdot 2$.)

8. For $x^2 + y^2 = 25$, $\frac{dy}{dx} = -\frac{x}{y}$. Find the slope at $(3, 4)$ (a decimal).

9. If $f(2) = 5$ and $f'(2) = 4$, find $(f^{-1})'(5)$.

10. For $y = \arctan(3x)$, $y' = \frac{3}{1 + 9x^2}$. Find y' at $x = 0$.

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Answers ·

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1. $f'(g(x)) \cdot g'(x)$

Outer derivative at the inner, times the inner derivative.

2. $2y \frac{dy}{dx}$

Chain rule: $2y$ times the inner derivative $\frac{dy}{dx}$.

3. $\frac{1}{f'(f^{-1}(x))}$

One over the original derivative at the matching input $f^{-1}(x)$.

4. $\frac{1}{\sqrt{1-x^2}}$

arcsin has the root form $\frac{1}{\sqrt{1-x^2}}$.

5. **product rule**

It is a product of x^2 and $\sin x \rightarrow$ product rule.

6. **8**

$$f'(2) = 6(2) - 4 = 8.$$

7. **6**

$$y' = 6(2x + 1)^2; \text{ at } 0: 6(1) = 6.$$

8. **-0.75**

$$-\frac{3}{4} = -0.75.$$

9. **0.25**

$$f^{-1}(5) = 2, \text{ so } (f^{-1})'(5) = \frac{1}{f'(2)} = \frac{1}{4}.$$

10. **3**

$$\text{At } x = 0: \frac{3}{1+0} = 3.$$