1. Limits and Continuity

Starter Quiz ·

AP Calculus BC Name: _____ Score: _____

1. The slope of a **secant line** between two points on a curve gives the...

   ☐ average rate of change over that interval

   ☐ exact value of the function

   ☐ area under the curve

   ☐ instantaneous rate of change

2. The notation $\lim_{x\to c} f(x) = L$ means that as x...

   ☐ is greater than c, $f(x)$ equals L

   ☐ equals c, the output is exactly L

   ☐ grows without bound, $f(x)$ reaches L

   ☐ gets close to c (without equalling it), the output heads toward L

3. A table gives $f(2.9) = 6.9$, $f(2.99) = 6.99$, $f(3.01) = 7.01$, $f(3.1) = 7.1$. Estimate $\lim_{x\to 3} f(x)$.

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4. Evaluate $\displaystyle\lim_{x\to 4} \frac{x^2 - 16}{x - 4}$.

   _____

5. Direct substitution into $\lim_{x\to 4} \dfrac{x-4}{x^2-16}$ gives $\frac{0}{0}$. What next?

   ☐ factor and cancel the common $(x - 4)$

   ☐ declare it does not exist

   ☐ report the limit as 0

   ☐ multiply by a conjugate

6. A graph shows the curve heading to height 3 from both sides of $x = 2$, with a filled dot at $(2, 7)$. Which statement matches?

   ☐ $\lim_{x\to 2} f(x) = 3$ and $f(2) = 7$

   ☐ the limit does not exist

   ☐ $\lim_{x\to 2} f(x) = 7$

   ☐ $f(2) = 3$

7. Select **all** conditions required for f to be continuous at c.

   ☐ f is a polynomial

- ☐ $\lim_{x \to c} f(x)$ exists
- ☐ $f(c)$ is defined
- ☐ $\lim_{x \to c} f(x) = f(c)$

Tick every correct option.

8. At the **left** endpoint a of a closed interval $[a, b]$, continuity requires...
   - ☐ nothing – endpoints are ignored
   - ☐ $f(a) = 0$
   - ☐ $\lim_{x \to a^-} f(x) = f(a)$
   - ☐ $\lim_{x \to a^+} f(x) = f(a)$ (right-continuity)

9. At what x-value does $f(x) = \dfrac{x+1}{x-4}$ have a vertical asymptote?

   _____

10. To apply the IVT on $[a, b]$ with target N, you need **all** of...
    - ☐ $f(a)$ and $f(b)$ known
    - ☐ f continuous on $[a, b]$
    - ☐ N between $f(a)$ and $f(b)$
    - ☐ f differentiable on $[a, b]$

    Tick every correct option.

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Answers ·

AP Calculus BC

1. **average rate of change over that interval**

   A secant joins two points, so its slope is the **average rate of change** over that interval.

2. **gets close to c (without equalling it), the output heads toward L**

   $x \to c$ means x **approaches** c without touching it, and $f(x)$ heads toward L.

3. **7**

   Both sides close in on 7, so the limit is 7.

4. **8**

   Factor: $\frac{(x-4)(x+4)}{x-4} = x + 4 \to 8$.

5. **factor and cancel the common $(x - 4)$**

   Polynomial over polynomial, both zero at $4 \to$ factor: $\frac{x-4}{(x-4)(x+4)} = \frac{1}{x+4} \to \frac{1}{8}$.

6. **$\lim_{x\to 2} f(x) = 3$ and $f(2) = 7$**

   The curve's approach gives the limit 3; the filled dot gives the separate value $f(2) = 7$.

7. **$\lim_{x\to c} f(x)$ exists; $f(c)$ is defined; $\lim_{x\to c} f(x) = f(c)$**

   The three conditions are defined, limit-exists, and equal. Being a polynomial is not required.

8. **$\lim_{x\to a^+} f(x) = f(a)$ (right-continuity)**

   You can only approach the left endpoint from the right, so right-continuity is required.

9. **4**

   Denominator zero at $x = 4$, numerator $= 5 \neq 0 \to$ asymptote $x = 4$.

10. **$f(a)$ and $f(b)$ known; f continuous on $[a, b]$; N between $f(a)$ and $f(b)$**

    IVT needs continuity and a target in range. Differentiability is **not** required.