

No formula sheet is supplied. BC assumes the whole of AB from memory and adds four areas that carry their own recall load: series with their convergence tests, parametric and polar calculus, the extra integration techniques, and the error bounds. This booklet covers the BC-only material and the conditions each test needs.

Integration techniques beyond AB

Integration by parts – $\int u dv = uv - \int v du$. Choose u so that differentiating it simplifies. For $\int x^2 e^x dx$ you apply it twice; for $\int e^x \sin x dx$ you apply it twice and solve for the original integral.

Partial fractions – Only for a proper rational function with distinct linear factors on this exam. Split, then each piece integrates to a logarithm.

Improper integrals – Rewrite the infinite limit as a limit of a definite integral and evaluate it. Writing the limit notation is itself a scored step; converging or diverging must then be stated.

Arc length – $L = \int_a^b \sqrt{1 + (dy/dx)^2} dx$, and for a parametric curve $L = \int \sqrt{(dx/dt)^2 + (dy/dt)^2} dt$.

Logistic growth – $\frac{dP}{dt} = kP \left(1 - \frac{P}{L}\right)$ with carrying capacity L . Growth is fastest at $P = L/2$, and $P \rightarrow L$ as $t \rightarrow \infty$ – both are asked in words far more often than the solution is asked for.

Euler's method – Step forward with $y_{n+1} = y_n + h f(x_n, y_n)$. Show the table of steps; the arithmetic alone earns little without it.

Parametric, vector and polar

Parametric derivatives – $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$, and the second derivative is $\frac{d}{dt} \left(\frac{dy}{dx} \right) \div \frac{dx}{dt}$ – not the ratio of the second derivatives, which is the standard trap.

Vector-valued motion – Position $\langle x(t), y(t) \rangle$; velocity is the derivative componentwise; SPEED is $\sqrt{(dx/dt)^2 + (dy/dt)^2}$, a scalar. Total distance is the integral of the speed.

Polar area – $A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$. Finding the correct limits – where the curve traces the region exactly once – carries more of the credit than the integral does.

Series – the tests and their conditions

Geometric – Converges exactly when $|r| < 1$, to $\frac{a}{1-r}$. The only common series whose sum you can state.

p-series $\sum 1/n^p$ – $\sum 1/n^p$ converges when $p > 1$. The harmonic series $p = 1$ diverges, and knowing that decides many comparison questions immediately.

nth-term test – If the terms do not tend to zero, the series diverges. It can NEVER prove convergence, and claiming it does is a lost point.

Ratio test – Compute $\lim |a_{n+1}/a_n|$: below 1 converges, above 1 diverges, exactly 1 is inconclusive and you must switch tests. This is the standard tool for a radius of convergence.

Comparison and limit comparison – Both require positive terms – say so. Name the series you are comparing with and why it converges or diverges.

Alternating series – Converges when the terms decrease in absolute value and tend to zero; both conditions must be stated. The error is then smaller than the first omitted term.

Absolute versus conditional – If $\sum |a_n|$ converges the series converges absolutely; if the series converges but $\sum |a_n|$ does not, it converges conditionally. The exam asks which, so answer with the word.

Taylor and Maclaurin

The general form – $\sum \frac{f^{(n)}(a)}{n!}(x-a)^n$. Building this from a table of derivative values is a recurring free-response part.

The four to memorise – $e^x = \sum \frac{x^n}{n!}$; $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$; $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$; $\frac{1}{1-x} = \sum x^n$ for $|x| < 1$. Every other series in the exam is built from these by substitution, differentiation or integration.

Interval of convergence – Use the ratio test for the radius, then TEST BOTH ENDPOINTS separately. The endpoints are where the marks are, and they are the step most often skipped.

Lagrange error bound – $|R_n(x)| \leq \frac{\max |f^{(n+1)}|}{(n+1)!} |x-a|^{n+1}$. State what bound you are using for the maximum derivative and why it is valid on the interval.

How BC free-response is scored

Name the test before applying it, and state its conditions. “By the alternating series test, since the terms decrease and tend to zero” is two scored statements; “it converges” is none.

For series questions, show the general term. A pattern written out as four terms with dots is accepted only when a general term accompanies it.

Set up every integral in full even when the calculator evaluates it, with correct limits and the variable of integration, then give the value to three decimal places.