10 Infinite series – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	**Write it 3 times** (English)		
sequence			
series			
geometric series			

Recall

You met geometric sequences earlier – each term multiplies by a fixed ratio:

- $2, 4, 8, 16, \ldots$ doubles each time.
- What if you add up **all** the terms? Can an infinite sum have a finite answer?

This sheet introduces infinite series. Each idea has a worked example.

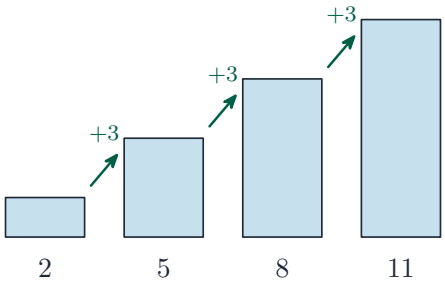
New ideas

■ 1 Sequence versus series

A **sequence** is a **list** of terms; a **series** is the **sum** of those terms.

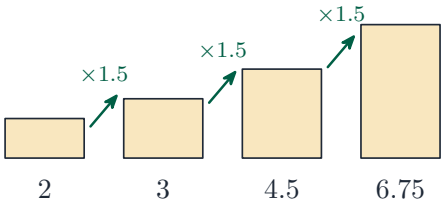
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arithmetic: add d each step



$u_n = a + (n-1)d$: equal steps

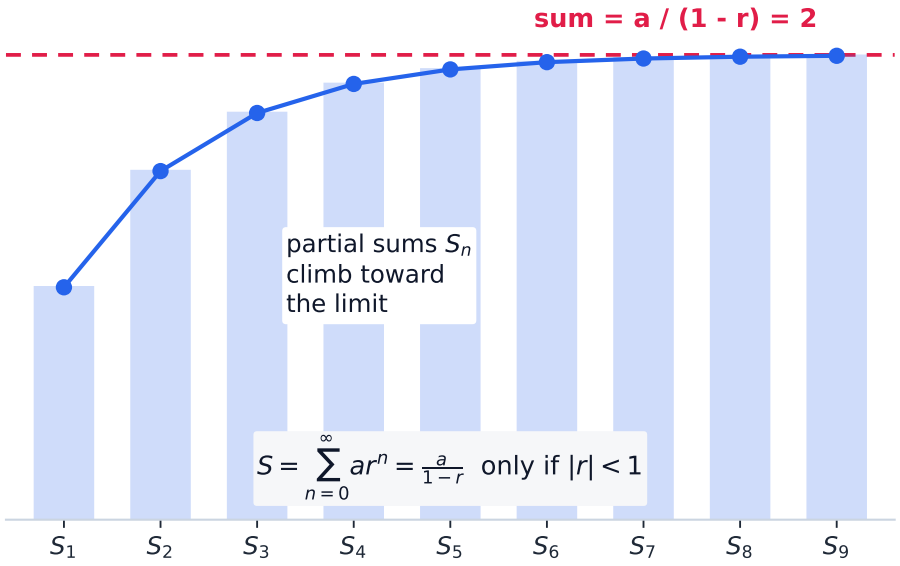
geometric: multiply by r each step



$u_n = ar^{n-1}$: growing steps

2 Geometric series

A **geometric series** adds terms that each multiply by a ratio r: $a + ar + ar^2 + \cdots$.



3 Infinite sum

If $|r| < 1$, the terms shrink toward zero and the infinite sum **converges** to a finite value:

$$S = \frac{a}{1 - r}.$$

Worked example. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ has $a = 1$, $r = \frac{1}{2}$, so $S = \dfrac{1}{1 - \frac{1}{2}} = 2$.

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■ 4 Converge or diverge

If $|r| \geq 1$ the terms do not shrink, so the sum grows without limit – it **diverges** (no finite answer).

Watch out: an infinite sum only has a finite value when the terms **shrink fast enough** ($|r| < 1$ for a geometric series). If the terms stay the same size or grow, the sum diverges.

Practice

Try these in order. The methods are all above.

2.1 State the difference between a sequence and a series. [2]

2.2 Write the formula for the sum of an infinite geometric series with $|r| < 1$. [1]

2.3 Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \cdots$ (with $a = 1$, $r = \frac{1}{3}$). [3]

2.4 Find the sum of $4 + 2 + 1 + \frac{1}{2} + \cdots$ (with $a = 4$, $r = \frac{1}{2}$). [2]

2.5 State whether $1 + 2 + 4 + 8 + \cdots$ converges, and why. [2]