

1.13 Removing Discontinuities

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS removable discontinuity • hole • limit • continuous
• jump

Objective

Build the skills to answer exam questions on **removing discontinuities**.

You must be able to:

- remove a **removable discontinuity** by redefining f at the hole

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Removing a discontinuity

A removable discontinuity (a hole) can be removed by defining $f(a) = \lim_{x \rightarrow a} f(x)$.

■ Example

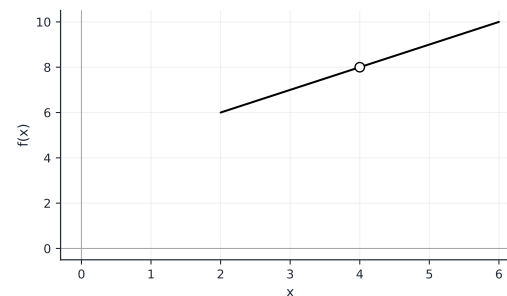
$f(x) = \frac{x^2 - 4}{x - 2}$ has a hole at $x = 2$; since the limit there is 4, setting $f(2) = 4$ makes f continuous.

■ Key knowledge: Repair a removable discontinuity

A removable discontinuity is repaired by assigning the existing finite limit. The simplified nearby formula supplies that limit but need not equal the original value at the missing input.

■ Worked example: Repair a removable discontinuity

For $f(x) = (x^2 - 16)/(x - 4)$ when $x \neq 4$, simplify to $x + 4$. The limit at 4 is 8, so defining $f(4) = 8$ gives a continuous extension. Defining $f(4) = 0$ would leave the same nearby limit but fail continuity.



2 Practice

2.1 State how to remove a removable discontinuity. [1]

2.2 For $f(x) = \frac{x^2 - 4}{x - 2}$, find the value that removes the hole at 2. [2]

2.3 State which kind of discontinuity can be removed. [1]

3 Exam-style questions

3.1 A removable discontinuity is removed by setting $f(a)$ equal to [1]

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- A 0
B the limit at a
C $f(a + 1)$
D ∞

3.2 Which discontinuity can be removed? [1]

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- A jump
B infinite
C removable
D none

3.3 $f(x) = \frac{x^2 - 9}{x - 3}$, with a hole at 3.

(a) Simplify f . [1]

(b) Find the limit at 3. [1]

(c) State $f(3)$ to remove the hole. [1]

3.4 (a) Find the value needed at 1 to extend $(x^2 - 1)/(x - 1)$ continuously. Show why. [2]

(b) Explain why the same single-point repair cannot remove a jump with left limit 2 and right limit 5. [2]