8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS rate in • rate out • accumulate • reservoir • interpret • trapezoidal sum

Objective

Build the skills to answer exam questions on **accumulation in applied contexts** : amount = start + rate in − rate out, interpreting integrals with units, tables and trapezoids, and the time of a maximum.

You must be able to:

- explain the meaning of a definite integral of a rate, with units
- find an amount from a starting value and a rate
- combine a rate in and a rate out
- find when an amount is greatest

1 Key knowledge and worked examples

■ Rates in and out

idea	formula or meaning
amount at time t	$A(t) = A(0) + \int_0^t \big(\text{rate in} - \text{rate out}\big)\,du$
meaning of $\int_a^b R(t)\,dt$	the **total amount** that flows from $t = a$ to $t = b$
units	(rate units) × (time units)
amount increasing	rate in > rate out
maximum amount	where rate in − rate out changes from + to −; check the endpoints too
from a table	trapezoids: $\frac{R_1+R_2}{2}\times$ width

Worked example 1: one rate. Water enters a tank at $R(t) = 20 + 2t$ liters per minute for 10 minutes.

1. Known: amount $= \int_0^{10} (20 + 2t)\,dt$.
2. Rule: antiderivative $20t + t^2$.
3. Numbers: $200 + 100$.
4. Result: 300 **liters**.

Worked example 2: a maximum. People enter at $E(t)$ and leave at $L(t)$.

1. Known: $N'(t) = E(t) - L(t)$.
2. Rule: N is greatest where $E - L$ changes from positive to negative.
3. Numbers: solve $E(t) = L(t)$.
4. Result: **compare N there with N at the endpoints**.

Answer shapes: a meaning sentence names the quantity, the time interval and the units.

2 Practice

2.1 Cars pass a toll gate at a rate of $r(t)$ cars per hour, t hours after 6 a.m. Explain the meaning of $\int_2^5 r(t)\,dt = 1200$. [2]

$\int_2^5 r(t)\,dt = 1200$

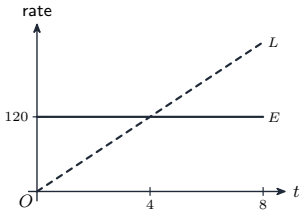
2.2 Rain falls into a barrel at $R(t) = 3\sqrt{t}$ liters per hour. At $t = 0$ the barrel holds 10 liters. How much water is in the barrel at $t = 4$? [3]

$R(t) = 3\sqrt{t}$ 10 L at $t = 0$

exercises.lol exercises.com.cn

3 Exam-style questions

3.1 People enter a park at $E(t) = 120$ people per hour and leave at $L(t) = 30t$ people per hour, for $0 \le t \le 8$. At $t = 0$ there are 50 people in the park. Find when the number of people is greatest, and that number. [3]



3.2 Water flows **out** of a reservoir at a rate $R(t)$ thousand liters per hour. Some values are given in the table.

(a) Estimate $R'(3.5)$. Give units. [2]

t (hours)	0	2	5	8
$R(t)$	10	14	20	16

(b) Use a trapezoidal sum with the subintervals in the table to estimate $\int_0^8 R(t)\,dt$. Give units. [3]

exercises.lol exercises.com.cn

(c) Use (b) to estimate the average rate of outflow for $0 \le t \le 8$, and explain its meaning. [3]

(d) At $t = 0$ the reservoir holds 500 thousand liters, and water also flows **in** at a constant 15 thousand liters per hour. Estimate the amount at $t = 8$. Is the amount increasing or decreasing at $t = 5$? Justify. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer, **B1** an independent mark.

Each answer shows the steps, then the **result**.

2.1

- the number of cars that pass the gate between 8 a.m. and 11 a.m.
- is **1200 cars**

2.2

- $\displaystyle\int_0^4 3\sqrt{t}\,dt = \left[2t^{3/2}\right]_0^4$
- $= 16$ liters
- $10 + 16 =$ **26 liters**

3.1

- $E - L = 120 - 30t$ changes from $+$ to $-$ at $t = 4$
- $N(4) = 50 + \displaystyle\int_0^4 (120 - 30t)\,dt = 50 + 480 - 240$
- **290** people at $\mathbf{t = 4}$; $N(8) = 50$ is smaller

3.2

(a) $\dfrac{20 - 14}{5 - 2} =$ **2** thousand liters per hour per hour

(b) $2 \cdot \frac{10+14}{2} + 3 \cdot \frac{14+20}{2} + 3 \cdot \frac{20+16}{2}$; $24 + 51 + 54 =$ **129** thousand liters

(c) $\frac{129}{8} \approx$ **16.1** thousand liters per hour; on average, about 16 100 liters leave each hour in these 8 hours

(d) $500 + 15(8) - 129 =$ **491** thousand liters; at $t = 5$: $15 - R(5) = 15 - 20 = -5$; **decreasing**

exercises.lol exercises.com.cn