7.7 Finding Particular Solutions Using Initial Conditions

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS initial condition

Objective

Build the skills to answer exam questions on **particular solutions from initial conditions** – pinning down the constant C.

You must be able to:

- find the **general solution** by separation of variables
- substitute the **initial condition** to solve for C
- write the **particular solution** and use it to predict a value

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

General, then particular

Solve $\dfrac{dy}{dx} = 2x$ with $y(0) = 3$. Integrate: $y = x^2 + C$ (general). Apply the condition $y(0) = 3$: $3 = 0 + C$, so $C = 3$. Particular solution: $y = x^2 + 3$.

Finding C after an exponential solution

$\dfrac{dy}{dt} = ky$ has general solution $y = Ae^{kt}$. If $y(0) = 5$, then $5 = Ae^0 = A$, so $A = 5$ and $y = 5e^{kt}$. The initial value **is** the constant A here.

Substituting a non-zero start

Solve $\dfrac{dy}{dx} = \dfrac{x}{y}$, $y(0) = 2$. Separating gives $y^2 = x^2 + C_1$. At $(0, 2)$: $4 = 0 + C_1$, so $y^2 = x^2 + 4$, i.e. $y = \sqrt{x^2 + 4}$ (positive root, to match $y(0) = 2 > 0$).

Using the particular solution

Once you have $y = \sqrt{x^2 + 4}$, predict any value: $y(3) = \sqrt{9 + 4} = \sqrt{13}$.

2 Practice

Now apply the methods above.

2.1 Solve $\dfrac{dy}{dx} = 4x$ with $y(1) = 5$. [3]

2.2 For $y = Ae^{2t}$ with $y(0) = 7$, find A. [1]

2.3 Solve $\dfrac{dy}{dx} = 3x^2$ with $y(0) = -1$. [3]

3 Exam-style questions

3.1 The general solution $y = x^2 + C$ passes through $(2, 10)$. Then $C =$ [1]

- **A** 2
- **B** 6
- **C** 10
- **D** 14

3.2 A population satisfies $\dfrac{dP}{dt} = 0.5P$ with $P(0) = 200$.

(a) Write the general solution. [2]

(b) Find the particular solution using the initial condition. [1]

(c) Predict P at $t = 4$. [2]

3.3 Solve $\dfrac{dy}{dx} = \dfrac{6x}{y}$ with $y(0) = 4$, giving y explicitly. [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $y = 2x^2 + C$; $5 = 2 + C \Rightarrow C = 3$; $y = 2x^2 + 3$

2.2

- $A = 7$

2.3

- $y = x^3 + C$; $-1 = 0 + C \Rightarrow C = -1$; $y = x^3 - 1$

3.1 B

- $10 = 4 + C \Rightarrow C = 6$

3.2

(a)

- $P = Ae^{0.5t}$

(b)

- $P(0) = A = 200$, so $P = 200e^{0.5t}$

(c)

- $P(4) = 200e^2 \approx 1478$

3.3

- $y\,dy = 6x\,dx$; $\frac{1}{2}y^2 = 3x^2 + C$; at $(0, 4)$: $8 = C$; $y = \sqrt{6x^2 + 16}$