6.6 Applying Properties of Definite Integrals

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS integral • definite integral

Objective

Build the skills to answer exam questions on the **properties of definite integrals** – combining and manipulating integral values.

You must be able to:

- use $\int_a^a f = 0$ and $\int_b^a f = -\int_a^b f$
- **split** an integral over adjacent intervals: $\int_a^c f = \int_a^b f + \int_b^c f$
- factor out constants and add integrands: $\int_a^b (cf + g) = c\int_a^b f + \int_a^b g$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

■ **Zero-width and reversing**

$\int_a^a f\,dx = 0$ (no width). Reversing the limits flips the sign: $\int_b^a f\,dx = -\int_a^b f\,dx$.

■ **Adding adjacent intervals**

Integrals join end-to-end: $\int_1^4 f = \int_1^3 f + \int_3^4 f$. If $\int_1^3 f = 5$ and $\int_1^4 f = 9$, then $\int_3^4 f = 9 - 5 = 4$.

■ **Linearity**

Constants come out and sums split:

$$\int_a^b \big(3f(x) + 2g(x)\big)\,dx = 3\int_a^b f + 2\int_a^b g.$$

If $\int_0^2 f = 4$ and $\int_0^2 g = 1$, then $\int_0^2 (3f + 2g) = 3(4) + 2(1) = 14$.

■ **Combining with a reversal**

Given $\int_2^5 f = 10$, find $\int_5^2 3f$. Reverse and factor: $\int_5^2 3f = 3\int_5^2 f = 3(-10) = -30$.

2 Practice

Now apply the methods above. You are given $\int_0^3 f = 6$, $\int_3^7 f = -2$, and $\int_0^3 g = 5$.

2.1 Find $\int_0^7 f\,dx$. [1]

2.2 Find $\int_3^0 f\,dx$. [1]

2.3 Find $\int_0^3 (2f + g)\,dx$. [2]

3 Exam-style questions

3.1 $\int_5^5 f(x)\,dx =$ [1]

☐ **A** $f(5)$
B 1
C 0
D undefined

3.2 You are given $\int_1^6 h = 12$ and $\int_1^4 h = 7$.

(a) Find $\int_4^6 h\,dx$. [2]

(b) Find $\int_6^1 h\,dx$. [1]

3.3 Given $\int_0^2 p = 8$ and $\int_0^2 q = -3$, find $\int_0^2 (5p - 4q)\,dx$. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\int_0^7 f = \int_0^3 f + \int_3^7 f = 6 + (-2) = 4$

2.2

- $\int_3^0 f = -\int_0^3 f = -6$

2.3

- $2(6) + 5 = 17$

3.1 C

- a zero-width interval gives 0

3.2

(a)

- $\int_4^6 h = \int_1^6 h - \int_1^4 h = 12 - 7 = 5$

(b)

- $\int_6^1 h = -12$

3.3

- $5(8) - 4(-3) = 40 + 12 = 52$