4.7 Using L'Hospital's Rule for Indeterminate Forms

Name: _____ Class: _____ Date: _____ **Total: 9 marks**

KEY WORDS l'hospital's rule • indeterminate form

Objective

Build the skills to answer exam questions on **L'Hospital's Rule for indeterminate forms**.

You must be able to:

- recognise an **indeterminate form** $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- apply **L'Hospital's Rule**

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ **L'Hospital's Rule**

For a limit of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,

$$\lim \frac{f}{g} = \lim \frac{f'}{g'}$$

(when the right-hand limit exists).

■ **Example**

$\lim_{x\to 0} \dfrac{\sin x}{x} = \lim_{x\to 0} \dfrac{\cos x}{1} = 1.$

2 Practice

2.1 State the forms L'Hospital's Rule applies to. [1]

2.2 Apply it to $\lim_{x\to 0} \dfrac{\sin x}{x}$. [2]

2.3 State what you do to the numerator and denominator. [1]

3 Exam-style questions

3.1 L'Hospital's Rule applies to [1]

- ☐ **A** any limit
- **B** $\frac{0}{0}$ or $\frac{\infty}{\infty}$
- **C** $\frac{1}{0}$
- **D** $\frac{5}{2}$

3.2 L'Hospital's Rule replaces $\dfrac{f}{g}$ with [1]

- ☐ **A** $f \cdot g$
- **B** $\dfrac{f'}{g'}$
- **C** $\dfrac{f'}{g}$
- **D** $\dfrac{f}{g'}$

3.3 $\lim_{x\to 0} \dfrac{e^x - 1}{x}$.

(a) Check the form. [1]

(b) Differentiate top and bottom. [1]

(c) Evaluate. [1]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{0}{0}$ and $\frac{\infty}{\infty}$

2.2

- $\lim_{x \to 0} \dfrac{\cos x}{1} = 1$

2.3

- differentiate each separately, then take the limit of the new quotient

3.1 B

- $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

3.2 B

- $\dfrac{f'}{g'}$.

3.3

(a)

- $\frac{0}{0}$

(b)

- $\dfrac{e^x}{1}$

(c)

- 1