# 10.9 Determining Absolute or Conditional Convergence

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

**KEY WORDS** converges conditionally • absolute convergence • converges • diverges • alternating series

## Objective

Build the skills to answer exam questions on **absolute and conditional convergence**.

**You must be able to:**

- test whether $\sum |a_n|$ converges (**absolute convergence** )
- classify a convergent series as absolutely or **conditionally convergent**
- use the standard example $\sum \frac{(-1)^n}{n}$

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

### Absolute convergence

$\sum a_n$ **converges absolutely** if $\sum |a_n|$ converges. Absolute convergence is the **stronger** property – it implies the series itself converges.

### Conditional convergence

If $\sum a_n$ converges but $\sum |a_n|$ diverges, the series **converges conditionally** – it relies on the cancellation of signs.

### The standard example

$\sum \frac{(-1)^n}{n}$ converges (alternating series test), but $\sum \left|\frac{(-1)^n}{n}\right| = \sum \frac{1}{n}$ diverges. So it is **conditionally** convergent.

### The routine

1. Test $\sum |a_n|$. If it converges → **absolute**. 2. If not, test $\sum a_n$ itself (e.g. alternating series). If it converges → **conditional**; if not → **divergent**.

## 2 Practice

Now apply the methods above.

**2.1** For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left|\frac{(-1)^n}{n^2}\right|$ converge? [1]

**2.2** Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional). [1]

**2.3** For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series? [2]

## 3 Exam-style questions

**3.1** A series is **conditionally** convergent if [1]

☐ **A** both $\sum a_n$ and $\sum |a_n|$ converge
**B** $\sum a_n$ converges but $\sum |a_n|$ diverges
**C** both diverge
**D** $\sum |a_n|$ converges but $\sum a_n$ diverges

**3.2** Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\sum \frac{1}{\sqrt{n}}$ for convergence. [2]

(b) Test the alternating series itself. [1]

(c) Classify the series. [1]

**3.3** Explain why absolute convergence is "stronger" than conditional convergence. [2]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- Yes – $\sum \frac{1}{n^2}$ is a convergent p-series

**2.2**

- Absolutely convergent

**2.3**

- $\sum \frac{1}{n}$ diverges; so $\sum \frac{(-1)^n}{n}$ is **conditionally** convergent

**3.1 B**

- converges, but not absolutely

**3.2**

(a)

- $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$, diverges

(b)

- Alternating series test: $b_n = \frac{1}{\sqrt{n}}$ decreasing $\to 0$, so it converges

(c)

- Conditionally convergent

**3.3**

- Absolute convergence guarantees the series converges no matter how the terms are arranged, and it implies ordinary convergence; conditional convergence depends on the sign pattern and can be broken by rearrangement