

10.9 Determining Absolute or Conditional Convergence

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS converges conditionally

• absolute convergence

• converges

• diverges

• alternating series

Objective

Build the skills to answer exam questions on **absolute and conditional convergence**.

You must be able to:

- test whether $\sum |a_n|$ converges (**absolute convergence**)
- classify a convergent series as absolutely or **conditionally convergent**
- use the standard example $\sum \frac{(-1)^n}{n}$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

■ Absolute convergence

$\sum a_n$ **converges absolutely** if $\sum |a_n|$ converges. Absolute convergence is the **stronger** property – it implies the series itself converges.

■ Conditional convergence

If $\sum a_n$ converges but $\sum |a_n|$ diverges, the series **converges conditionally** – it relies on the cancellation of signs.

■ The standard example

$\sum \frac{(-1)^n}{n}$ converges (alternating series test), but $\sum \left| \frac{(-1)^n}{n} \right| = \sum \frac{1}{n}$ diverges. So it is **conditionally** convergent.

■ The routine

1. Test $\sum |a_n|$. If it converges $\rightarrow$ **absolute**.
2. If not, test $\sum a_n$ itself (e.g. alternating series). If it converges $\rightarrow$ **conditional**; if not $\rightarrow$ **divergent**.

2 Practice

Now apply the methods above.

2.1 For $\sum \frac{(-1)^n}{n^2}$, does $\sum \left| \frac{(-1)^n}{n^2} \right|$ converge? [1]

2.2 Classify $\sum \frac{(-1)^n}{n^2}$ (absolute / conditional). [1]

2.3 For $\sum \frac{(-1)^n}{n}$, does $\sum \frac{1}{n}$ converge? What does that make the series? [2]

3 Exam-style questions

3.1 A series is **conditionally** convergent if [1]

- ☐ **A** both $\sum a_n$ and $\sum |a_n|$ converge
B $\sum a_n$ converges but $\sum |a_n|$ diverges
C both diverge
D $\sum |a_n|$ converges but $\sum a_n$ diverges

3.2 Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

(a) Test $\sum \frac{1}{\sqrt{n}}$ for convergence. [2]

(b) Test the alternating series itself. [1]

(c) Classify the series. [1]

3.3 Explain why absolute convergence is “stronger” than conditional convergence. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- Yes – $\sum \frac{1}{n^2}$ is a convergent p-series

2.2

- Absolutely convergent

2.3

- $\sum \frac{1}{n}$ diverges; so $\sum \frac{(-1)^n}{n}$ is **conditionally** convergent

3.1 B

- converges, but not absolutely

3.2

(a)

- $\sum \frac{1}{\sqrt{n}}$ is a p-series with $p = \frac{1}{2} \leq 1$, diverges

(b)

- Alternating series test: $b_n = \frac{1}{\sqrt{n}}$ decreasing $\rightarrow 0$, so it converges

(c)

- Conditionally convergent

3.3

- Absolute convergence guarantees the series converges no matter how the terms are arranged, and it implies ordinary convergence; conditional convergence depends on the sign pattern and can be broken by rearrangement