

# 10.2 Working with Geometric Series

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 13 marks

**KEY WORDS** ratio • converges • geometric series • diverges

## Objective

Build the skills to answer exam questions on **geometric series** – the one series you can sum exactly.

**You must be able to:**

- identify the first term  $a$  and common **ratio**  $r$
- apply the convergence test  $|r| < 1$
- compute the sum  $\frac{a}{1-r}$

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ The geometric sum

A geometric series  $\sum_{n=0}^{\infty} ar^n$  converges **exactly when**  $|r| < 1$ , and then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}.$$

### ■ Reading a and r

For  $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n$ :  $a = 3$ ,  $r = \frac{1}{2}$ . Since  $|r| < 1$ ,  $\text{sum} = \frac{3}{1 - \frac{1}{2}} = 6$ .

### ■ Watch the starting index

If the sum starts at  $n = 1$ , the first term is  $ar$ , not  $a$ .  $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1/3}{1 - 1/3} = \frac{1/3}{2/3} = \frac{1}{2}$ .

## ■ Divergence

If  $|r| \geq 1$  the terms do not shrink, so the series diverges.  $\sum 2^n$  has  $r = 2$ , diverges.

## 2 Practice

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Now apply the methods above.

**2.1** State  $a$  and  $r$  for  $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$ . [2]

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**2.2** Find the sum of  $\sum_{n=0}^{\infty} 5 \left(\frac{1}{3}\right)^n$ . [2]

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**2.3** Does  $\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$  converge? Give a reason. [1]

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## 3 Exam-style questions

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**3.1** A geometric series  $\sum ar^n$  converges when [1]

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**A**  $r > 0$

**B**  $|r| < 1$

**C**  $|r| > 1$

**D**  $a < 1$

**3.2** Consider  $\sum_{n=0}^{\infty} 6 \left(-\frac{1}{2}\right)^n$ .

(a) State  $a$  and  $r$ . [2]

(b) Find the sum. [2]

**3.3** Find the sum of  $\sum_{n=1}^{\infty} 4 \left(\frac{2}{5}\right)^n$  (note the index starts at 1). [3]

## Solutions

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Each answer shows the steps, then the **result**.

### 2.1

- $a = 5, r = \frac{1}{3}$

### 2.2

- $\frac{5}{1 - 1/3} = \frac{5}{2/3} = \frac{15}{2}$

### 2.3

- No —  $|r| = \frac{5}{4} > 1$ , so it diverges

### 3.1 B

- converges iff  $|r| < 1$

### 3.2

(a)

- $a = 6, r = -\frac{1}{2}$

(b)

- $\frac{6}{1 - (-\frac{1}{2})} = \frac{6}{3/2} = 4$

### 3.3

- First term ( $n = 1$ ) is  $4 \cdot \frac{2}{5} = \frac{8}{5}$ , ratio  $\frac{2}{5}$ ;  $\text{sum} = \frac{8/5}{1 - 2/5} = \frac{8/5}{3/5} = \frac{8}{3}$