

# 10.14 Finding Taylor or Maclaurin Series for a Function

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 12 marks

KEY WORDS geometric series

## Objective

Build the skills to answer exam questions on **finding Taylor and Maclaurin series**.

**You must be able to:**

- recall the key **Maclaurin series** for  $e^x$ ,  $\sin x$ ,  $\cos x$ ,  $\frac{1}{1-x}$
- build new series by **substitution** into a known one
- write a series in summation form

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later – follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ The must-know Maclaurin series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n.$$

### ■ Building by substitution

For  $e^{x^2}$ , substitute  $x^2$  into the  $e^x$  series:

$$e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}.$$

### ■ Another substitution

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n.$$

■ Writing out terms

$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$ . Being able to write the first few terms **and** the general summation form is expected.

## 2 Practice

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Now apply the methods above.

**2.1** Write the Maclaurin series for  $e^x$  (summation form). [1]

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**2.2** Write the first three nonzero terms of  $\cos x$ . [2]

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**2.3** Use substitution to write the Maclaurin series for  $e^{-x}$ . [2]

## 3 Exam-style questions

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**3.1** The Maclaurin series for  $\frac{1}{1-x}$  is [1]

- ☐ A  $\sum (-1)^n x^n$
- ☐ B  $\sum x^n$
- ☐ C  $\sum \frac{x^n}{n!}$
- ☐ D  $\sum nx^n$

**3.2** Find the Maclaurin series for  $\frac{1}{1+x^2}$  (substitute into the geometric series). [3]

**3.3** Find the first three nonzero terms of the Maclaurin series for  $x \sin x$ . [3]

## Solutions

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Each answer shows the steps, then the **result**.

**2.1**

$$\bullet \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

**2.2**

$$\bullet 1 - \frac{x^2}{2} + \frac{x^4}{24}$$

**2.3**

$$\bullet \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

**3.1 B**

$$\bullet \frac{1}{1-x} = \sum x^n$$

**3.2**

$$\bullet \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

**3.3**

$$\bullet \sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots; \text{ multiply by } x: x^2 - \frac{x^4}{6} + \frac{x^6}{120} - \cdots$$