AP CALCULUS BC • EXERCISE SHEET

1.8 Determining Limits Using the Squeeze Theorem

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS squeeze theorem • limit

Objective

Build the skills to answer exam questions on **the Squeeze Theorem**.

You must be able to:

- state the **Squeeze Theorem**
- apply it when a function is bounded between two others with equal limits

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

The Squeeze Theorem

If $g(x) \le f(x) \le h(x)$ near a and

$$\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L,$$

then $\lim_{x\to a} f(x) = L$.

Example

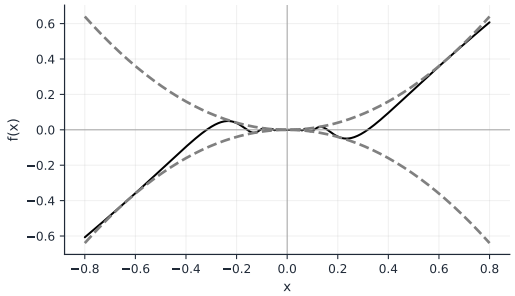
Since $-x^2 \le x^2 \sin\frac{1}{x} \le x^2$ and both bounds $\to 0$ at 0, the middle limit is 0.

Key knowledge: Check the squeeze hypotheses

A squeeze argument needs the bounding inequalities to hold near the target and both bounds to approach the same value. The middle function need not have a value at the target.

Worked example: Check the squeeze hypotheses

Because $-1 \le \sin(1/x) \le 1$, for $x \ne 0$ we have $-x^2 \le x^2\sin(1/x) \le x^2$. Both bounding functions approach 0, so the squeezed function also approaches 0. The oscillation alone does not prevent a limit when its amplitude shrinks to zero.



2 Practice

2.1 State the Squeeze Theorem. [1]

2.2 State what must be true of the two bounding limits. [1]

2.3 If $-x^2 \le f(x) \le x^2$ and both bounds $\to 0$ at 0, state $\lim_{x\to 0} f(x)$. [2]

3 Exam-style questions

3.1 The Squeeze Theorem requires $g \le f \le h$ and [1]

☐ **A** $\lim g = \lim h$
B $g = h$ everywhere
C $f = 0$
D $h = 0$

3.2 If both bounds approach 3, then $\lim f$ is [1]

- **A** 0
- **B** 3
- **C** 6
- **D** undefined

3.3 $-x^2 \leq f(x) \leq x^2$ near 0.

(a) State $\lim_{x\to 0}(-x^2)$. [1]

(b) State $\lim_{x\to 0} x^2$. [1]

(c) State $\lim_{x\to 0} f(x)$. [1]

3.4 For all sufficiently small non-zero x, $5 - x^2 \leq h(x) \leq 5 + x^2$. (a) Find the limits of both bounds. [2]

(b) Find the limit of h and name the conditions that justify your conclusion. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- if $g \leq f \leq h$ near a and g, h have the same limit L at a, then f also has limit L

2.2

- they must be equal (both equal L)

2.3

- $\lim_{x \to 0} f(x) = 0$

3.1 A

- $\lim g = \lim h$.

3.2 B

- 3.

3.3

(a)

- 0

(b)

- 0

(c)

- 0

3.4

(a) Each bound approaches 5.

(b) $\lim_{x \to 0} h(x) = 5$; the nearby inequalities and common bound limit satisfy the squeeze theorem.