1.7 Selecting Procedures for Determining Limits

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS vertical asymptote • indeterminate form • limit • direct substitution • one-sided limits

Objective

Build the skills to answer exam questions on **selecting procedures for determining limits**.

You must be able to:

- choose the right method (substitution, factoring, one-sided) for a limit
- interpret what $\frac{0}{0}$ and $\frac{\text{nonzero}}{0}$ signal

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Choosing a method

- try **direct substitution** first;
- $\frac{0}{0}$ (an **indeterminate form**) $\rightarrow$ factor and cancel, or rationalize;
- $\frac{\text{nonzero}}{0}$ $\rightarrow$ check for a vertical asymptote and use one-sided limits.

Example

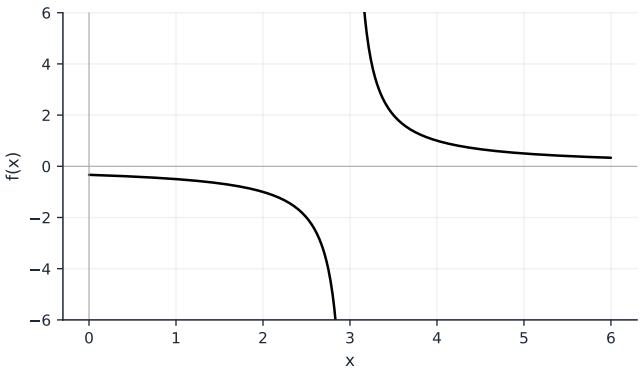
$\lim_{x\to 2} \dfrac{x-2}{x^2-4}$: substitution gives $\frac{0}{0}$, so cancel to $\dfrac{1}{x+2} = \frac{1}{4}$.

Key knowledge: Choose a limit method

First test substitution. A finite value needs no cancellation; 0/0 may need algebra; a non-zero numerator over a vanishing denominator needs sign analysis.

■ **Worked example: Choose a limit method**

The expression $(x^2 - 9)/(x - 3)$ needs factoring at 3 and has limit 6. The expression $1/(x - 3)$ instead tends to negative infinity from the left and positive infinity from the right. Cancelling is impossible in the second expression, and there is no finite two-sided limit.



2 Practice

2.1 State the first method to try. [1]

2.2 State what a $\frac{0}{0}$ result signals. [1]

2.3 State what a $\frac{\text{nonzero}}{0}$ result signals. [1]

3 Exam-style questions

3.1 The first thing to try for a limit is [1]

- ☐ **A** factoring
- **B** direct substitution
- **C** rationalizing
- **D** a table

3.2 A $\frac{0}{0}$ form means [1]

- **A** the limit is 0
- **B** more algebra is needed
- **C** the limit is ∞
- **D** no limit exists

3.3 $\lim_{x\to 2} \dfrac{x-2}{x^2-4}$.

(a) Substitute to see the form. [1]

(b) Factor the denominator. [1]

(c) Simplify and evaluate. [1]

3.4 (a) Find $\lim_{x\to 1}(x^2-1)/(x-1)$ and name your method. [2]

(b) Describe both one-sided limits of $1/(x-1)$ at 1 and decide whether a two-sided limit exists. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- direct substitution

2.2

- that more algebra (factoring/rationalizing) is needed

2.3

- a possible vertical asymptote – check one-sided limits

3.1 B

- direct substitution.

3.2 B

- more algebra is needed.

3.3

(a)

- $\frac{0}{0}$

(b)

- $(x-2)(x+2)$

(c)

- $\dfrac{1}{x+2} = \frac{1}{4}$

3.4

(a) Factor and cancel $x-1$ for $x \neq 1$; the limit of $x+1$ is 2.

(b) Left: $-\infty$; right: $+\infty$; no two-sided limit exists because the two sides do not agree.