

1.6 Determining Limits Using Algebraic Manipulation

Name: _____ Class: _____ Date: _____

Total: 13 marks**KEY WORDS** factor and cancel

• indeterminate form

• limit

• direct substitution

Objective

Build the skills to answer exam questions on **determining limits using algebraic manipulation**.

You must be able to:

- recognise an **indeterminate form** $\frac{0}{0}$
- factor and cancel to evaluate the limit

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Algebraic manipulation

When direct substitution gives $\frac{0}{0}$, factor and cancel the common factor, then substitute.

■ Example

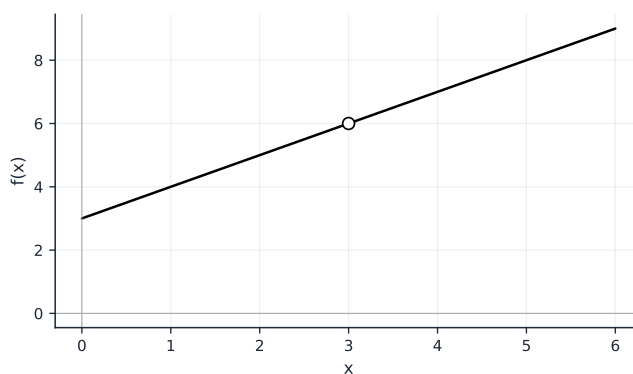
$$\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \lim_{x \rightarrow 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x + 2) = 4.$$

■ Key knowledge: Remove an indeterminate form

The form $0/0$ identifies a method problem, not a numerical answer. Rewrite the expression for nearby non-target inputs, then evaluate the simplified limit.

■ Worked example: Remove an indeterminate form

For $f(x) = (x^2 - 9)/(x - 3)$, direct substitution at 3 gives $0/0$. Factor and cancel for $x \neq 3$: $f(x) = (x - 3)(x + 3)/(x - 3) = x + 3$. Therefore $\lim_{x \rightarrow 3} f(x) = 6$. Cancellation preserves nearby values; it does not make the original expression defined at 3.



2 Practice

2.1 State what to do when substitution gives $\frac{0}{0}$. [1]

2.2 Simplify $\frac{x^2 - 9}{x - 3}$. [1]

2.3 Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$. [2]

3 Exam-style questions

3.1 If direct substitution gives $\frac{0}{0}$, you should [1]

- ☐ **A** conclude the limit is 0
☐ **B** factor and cancel
☐ **C** stop – no limit exists
☐ **D** divide by x

3.2 $\frac{x^2 - 1}{x - 1}$ simplifies to [1]

- ☐ **A** $x - 1$
☐ **B** $x + 1$
☐ **C** $x^2 + 1$
☐ **D** 1

3.3 $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}.$

(a) Factor the numerator. [1]

(b) Cancel $(x - 1)$. [1]

(c) Substitute $x = 1$. [1]

3.4 (a) Rationalise $(\sqrt{x + 4} - 2)/x$ for $x \neq 0$. [2]

(b) Hence find its limit as $x \rightarrow 0$ and explain why direct substitution into the original expression is insufficient. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- factor and cancel the common factor, then substitute

2.2

- $x + 3$ (for $x \neq 3$)

2.3

- $\lim_{x \rightarrow 3}(x + 3) = 6$

3.1 B

- factor and cancel.

3.2 B

- $x + 1$.

3.3

(a)

- $(x - 1)(x + 1)$

(b)

- $x + 1$

(c)

- 2

3.4

(a) Multiply by the conjugate: $(\sqrt{x+4} - 2)/x = 1/(\sqrt{x+4} + 2)$ for $x \neq 0$.

(b) The limit is $1/(2+2) = 1/4$; direct substitution into the original expression gives the indeterminate form $0/0$, not its limit.