1.5 Determining Limits Using Algebraic Properties of Limits

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS continuous • direct substitution • limit laws • limit

Objective

Build the skills to answer exam questions on **determining limits using algebraic properties of limits**.

You must be able to:

- apply the **limit laws** (sum, product, constant multiple)
- use **direct substitution** for continuous functions

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Limit laws

Limits distribute over sums, differences, products, quotients (denominator $\neq 0$), and constant multiples. For a continuous function, $\lim_{x\to a} f(x) = f(a)$ (**direct substitution**).

■ Example

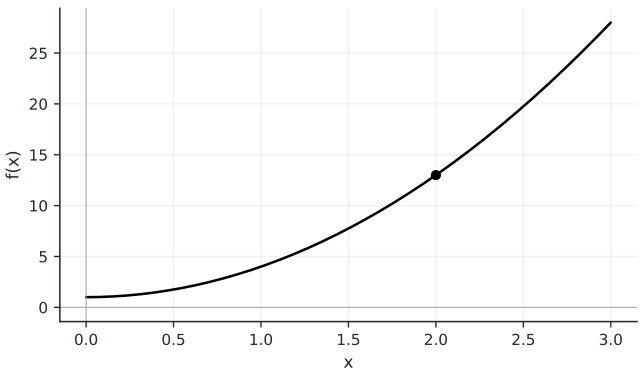
$\lim_{x\to 3}(x^2 - 1) = 3^2 - 1 = 8$.

■ Key knowledge: Apply limit laws with conditions

Limit laws combine existing finite limits. For a quotient, the limiting denominator must be non-zero before the quotient law can be applied.

■ Worked example: Apply limit laws with conditions

For the graph choose $f(x) = x$ and $g(x) = -1$ near $x = 2$. Given $\lim f(x) = 2$ and $\lim g(x) = -1$ at the same input, $\lim[3f(x)^2 - g(x)] = 3(2)^2 - (-1) = 13$. Also $\lim f(x)/g(x) = 2/(-1) = -2$, because the denominator limit is not zero. A zero denominator would require further analysis rather than this substitution.



2 Practice

2.1 State the limit of a sum $\lim(f+g)$. [1]

2.2 Evaluate $\lim_{x\to 2}(3x+1)$ by substitution. [2]

2.3 State when direct substitution works. [1]

3 Exam-style questions

3.1 $\lim(f+g)$ equals [1]

- **A** $\lim f \cdot \lim g$
- **B** $\lim f + \lim g$
- **C** $\lim f - \lim g$
- **D** $f(a)$

3.2 For a continuous function, $\lim_{x\to a} f(x)$ equals [1]

- **A** 0
- **B** $f(a)$
- **C** ∞
- **D** a

3.3 Evaluate $\lim_{x \to 3}(x^2 - 1)$.

(a) Substitute $x = 3$. [1]

(b) Compute 3^2. [1]

(c) State the limit. [1]

3.4 At $x = a$, $f(x) \to 3$ and $g(x) \to 2$. (a) Find the limit of $f(x) - g(x)$ and explain why a quotient with this denominator can use the quotient law. [2]

(b) Find $\lim_{x \to a}[2f(x) + g(x)]/[f(x) - g(x)]$. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\lim f + \lim g$

2.2

- $3(2) + 1 = 7$

2.3

- when the function is continuous at a

3.1 B

- $\lim f + \lim g$.

3.2 B

- $f(a)$.

3.3

(a)

- $3^2 - 1$

(b)

- 9

(c)

- 8

3.4

(a) The denominator limit is $3 - 2 = 1$, which is non-zero.

(b) The numerator limit is $2(3) + 2 = 8$; the quotient limit is $8/1 = 8$.