

1.3 Estimating Limit Values from Graphs

Name: _____ Class: _____ Date: _____

Total: 13 marks**KEY WORDS** one-sided limits

• hole

• limit

• left-hand limit

• right-hand limit

Objective

Build the skills to answer exam questions on **estimating limit values from graphs**.

You must be able to:

- read **one-sided limits** from a graph
- decide whether the two-sided limit exists

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Limits from graphs

Read the y -value the graph approaches from the left ($\lim_{x \rightarrow a^-}$) and from the right ($\lim_{x \rightarrow a^+}$).

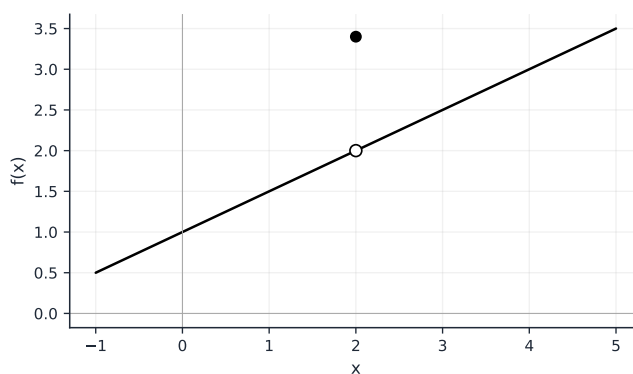
The two-sided limit $\lim_{x \rightarrow a} f(x)$ exists only if **both** one-sided limits equal the same L – even if there is a **hole** at a .

■ Key knowledge: Read approaching heights on a graph

Follow each curve toward the input from its own side. An open point gives an approaching height; a filled point gives the actual function value. Different one-sided limits rule out a two-sided limit.

■ Worked example: Read approaching heights on a graph

In the worked graph, the line approaches height 2 from both sides of $x = 2$, while the filled point is at 3.4. Hence both one-sided limits and the two-sided limit are 2, but the value at 2 is 3.4. Do not use the filled point as a substitute for tracing the curve.



2 Practice

2.1 State when a two-sided limit exists. [1]

2.2 State the notation for a left-hand limit. [1]

2.3 If the limit from the left is 4 and from the right is 4, state the two-sided limit. [2]

3 Exam-style questions

3.1 A two-sided limit exists if and only if [1]

- ☐ **A** $f(a)$ is defined
B the left and right limits are equal
C the graph is a straight line
D the limit is 0

3.2 A hole in the graph at $x = a$ means [1]

- ☐ **A** the limit cannot exist
B the limit may still exist
C f is linear
D $a = 0$

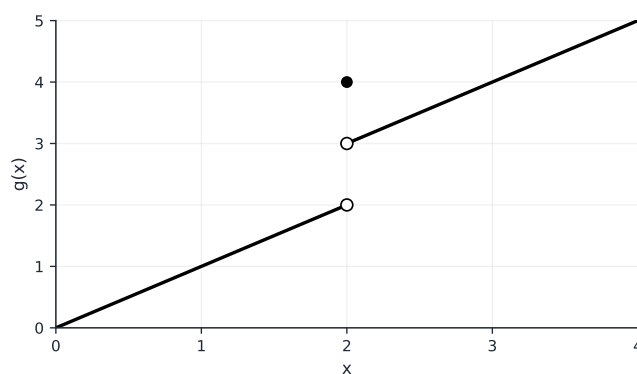
3.3 A graph approaches 5 from the left and 3 from the right at $x = 2$.

(a) State the left-hand limit. [1]

(b) State the right-hand limit. [1]

(c) State whether the two-sided limit exists. [1]

3.4 The graph defines g near $x = 2$.



(a) State its left- and right-hand limits at 2. [2]

(b) Does the two-sided limit exist? Explain using the graph. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- when the left-hand and right-hand limits are equal

2.2

- $\lim_{x \rightarrow a^-} f(x)$

2.3

- the two-sided limit is 4

3.1 B

- the left and right limits are equal.

3.2 B

- the limit may still exist.

3.3

(a)

- 5

(b)

- 3

(c)

- no, the one-sided limits differ

3.4

(a) The left-hand limit is 2 and the right-hand limit is 3.

(b) The two-sided limit does not exist, because the one-sided approaching heights differ. The filled point at 4 does not resolve that mismatch.