1.2 Defining Limits and Using Limit Notation

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS limit notation • arbitrarily • limit • approaches

Objective

Build the skills to answer exam questions on **defining limits and using limit notation**.

You must be able to:

- read and write **limit notation** $\lim_{x\to a} f(x) = L$
- explain that a limit need not equal $f(a)$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

Limit notation

$$\lim_{x\to a} f(x) = L$$

means $f(x)$ gets arbitrarily close to L as x approaches a from **both sides** – regardless of the value (or existence) of $f(a)$.

Example

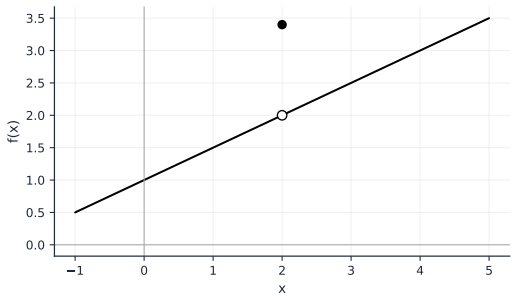
$f(x) = x + 1$: $\lim_{x\to 2} f(x) = 3$, which here also equals $f(2) = 3$.

Key knowledge: Separate the limit from the value

A limit describes nearby outputs. Changing the output at one point does not change a limit there, provided the nearby function stays the same.

Worked example: Separate the limit from the value

Take $f(x) = x/2 + 1$ for $x \neq 2$, and $f(2) = 3.4$. Approaching 2 from either side gives $\lim_{x\to 2} f(x) = 2$. The filled point gives $f(2) = 3.4$. Since these differ, the function is not continuous at 2 even though the two-sided limit exists.



2 Practice

2.1 State what $\lim_{x\to a} f(x) = L$ means. [1]

2.2 State whether the limit must equal $f(a)$. [1]

2.3 For $f(x) = 2x$, state $\lim_{x\to 3} f(x)$. [2]

3 Exam-style questions

3.1 $\lim_{x\to a} f(x) = L$ means $f(x)$ approaches [1]

- ☐ **A** a
- **B** L as x approaches a
- **C** infinity
- **D** $f(a)$ always

3.2 The value of a limit [1]

- ☐ **A** always equals $f(a)$
- **B** may differ from $f(a)$
- **C** is always 0
- **D** never exists

3.3 $f(x) = x + 1$.

(a) State $\lim_{x \to 2} f(x)$. [1]

(b) State $f(2)$. [1]

(c) State whether they agree. [1]

3.4 Let $g(x) = x + 3$ for $x \neq 1$, and $g(1) = 8$. (a) Find the left- and right-hand limits at 1. [2]

(b) State the two-sided limit and explain whether g is continuous at 1. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $f(x)$ gets arbitrarily close to L as x approaches a

2.2

- no – the limit may differ from $f(a)$

2.3

- $\lim_{x\to 3} 2x = 6$

3.1 B

- L as x approaches a.

3.2 B

- may differ from $f(a)$.

3.3

(a)

- 3

(b)

- 3

(c)

- yes

3.4

(a) Both one-sided limits are 4.

(b) The two-sided limit is 4; $g(1) = 8 \neq 4$, so continuity fails.