

# 1.14 Connecting Infinite Limits and Vertical Asymptotes

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_ **Total: 13 marks**

**KEY WORDS** vertical asymptote • infinite limit • limit  
• horizontal asymptote • rational function

## Objective

Build the skills to answer exam questions on **connecting infinite limits and vertical asymptotes**.

You must be able to:

- relate an **infinite limit** at  $a$  to a **vertical asymptote**  $x = a$

## 1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

### ■ Infinite limits

If  $f(x) \rightarrow \pm\infty$  as  $x \rightarrow a$ , the line  $x = a$  is a **vertical asymptote**. This happens where the denominator  $\rightarrow 0$  but the numerator does not.

### ■ Example

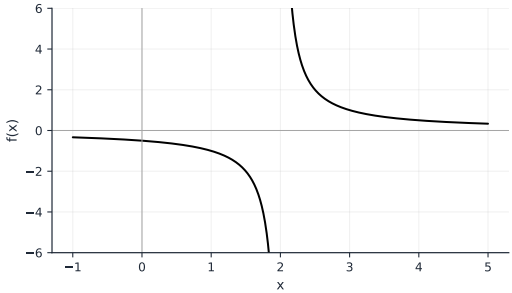
$f(x) = \frac{1}{x-2}$  has a vertical asymptote at  $x = 2$ ; as  $x \rightarrow 2^+$ ,  $f(x) \rightarrow +\infty$ .

### ■ Key knowledge: Use signs near an asymptote

An infinite limit describes unbounded outputs. For a simple reciprocal, determine the denominator sign on each side; opposite signs can give different infinite one-sided limits.

### ■ Worked example: Use signs near an asymptote

For  $f(x) = 1/(x-2)$ , the denominator is a small negative number from the left and a small positive number from the right. Hence the left limit is  $-\infty$  and the right limit is  $+\infty$ . The line  $x = 2$  is a vertical asymptote; there is no common two-sided limit.



## 2 Practice

2.1 State what an infinite limit at  $a$  indicates. [1]

2.2 For  $f(x) = \frac{1}{x}$ , state the vertical asymptote. [1]

2.3 State where a rational function has a vertical asymptote. [2]

## 3 Exam-style questions

3.1 If  $\lim_{x \rightarrow a} f(x) = \infty$ , then  $x = a$  is a [1]

- ☐ A horizontal asymptote  
☐ B vertical asymptote  
☐ C hole  
☐ D root

3.2  $f(x) = \frac{1}{x-2}$  has a vertical asymptote at [1]

- ☐ A  $x = 0$   
☐ B  $x = 2$   
☐ C  $y = 2$   
☐ D  $x = -2$

**3.3**  $f(x) = \frac{1}{x-5}$ .

(a) Where is the denominator 0? [1]

(b) State the vertical asymptote. [1]

(c) State the behaviour of  $\lim_{x \rightarrow 5^+} f(x)$ . [1]

**3.4** For  $g(x) = (x+1)/(x-3)$ , (a) find each one-sided limit as  $x \rightarrow 3$ . [2]

(b) Name the vertical asymptote and justify the signs using numerator and denominator. [2]

## Solutions

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Each answer shows the steps, then the **result**.

### 2.1

- the graph has a vertical asymptote there

### 2.2

- $x = 0$

### 2.3

- where the denominator is 0 but the numerator is not

### 3.1 B

- vertical asymptote.

### 3.2 B

- $x = 2$ .

### 3.3

(a)

- $x = 5$

(b)

- $x = 5$

(c)

- $+\infty$

### 3.4

(a) Left  $-\infty$ , right  $+\infty$ .

(b) The asymptote is  $x = 3$ ; the numerator approaches positive 4 while the denominator changes from small negative to small positive.