

1.14 Connecting Infinite Limits and Vertical Asymptotes

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS vertical asymptote • infinite limit • limit
• horizontal asymptote • rational function

Objective

Build the skills to answer exam questions on **connecting infinite limits and vertical asymptotes**.

You must be able to:

- relate an **infinite limit** at a to a **vertical asymptote** $x = a$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Infinite limits

If $f(x) \rightarrow \pm\infty$ as $x \rightarrow a$, the line $x = a$ is a **vertical asymptote**. This happens where the denominator $\rightarrow 0$ but the numerator does not.

■ Example

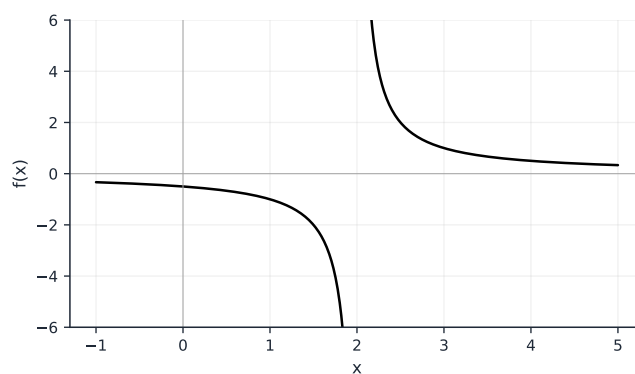
$f(x) = \frac{1}{x-2}$ has a vertical asymptote at $x = 2$; as $x \rightarrow 2^+$, $f(x) \rightarrow +\infty$.

■ Key knowledge: Use signs near an asymptote

An infinite limit describes unbounded outputs. For a simple reciprocal, determine the denominator sign on each side; opposite signs can give different infinite one-sided limits.

■ Worked example: Use signs near an asymptote

For $f(x) = 1/(x-2)$, the denominator is a small negative number from the left and a small positive number from the right. Hence the left limit is $-\infty$ and the right limit is $+\infty$. The line $x = 2$ is a vertical asymptote; there is no common two-sided limit.



2 Practice

2.1 State what an infinite limit at a indicates. [1]

2.2 For $f(x) = \frac{1}{x}$, state the vertical asymptote. [1]

2.3 State where a rational function has a vertical asymptote. [2]

3 Exam-style questions

3.1 If $\lim_{x \rightarrow a} f(x) = \infty$, then $x = a$ is a [1]

- ☐ **A** horizontal asymptote
☐ **B** vertical asymptote
☐ **C** hole
☐ **D** root

3.2 $f(x) = \frac{1}{x-2}$ has a vertical asymptote at [1]

- ☐ **A** $x = 0$
☐ **B** $x = 2$
☐ **C** $y = 2$
☐ **D** $x = -2$

3.3 $f(x) = \frac{1}{x-5}$.

- (a) Where is the denominator 0? [1]
- (b) State the vertical asymptote. [1]
- (c) State the behaviour of $\lim_{x \rightarrow 5^+} f(x)$. [1]

3.4 For $g(x) = (x+1)/(x-3)$, (a) find each one-sided limit as $x \rightarrow 3$. [2]

(b) Name the vertical asymptote and justify the signs using numerator and denominator. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- the graph has a vertical asymptote there

2.2

- $x = 0$

2.3

- where the denominator is 0 but the numerator is not

3.1 B

- vertical asymptote.

3.2 B

- $x = 2$.

3.3

(a)

- $x = 5$

(b)

- $x = 5$

(c)

- $+\infty$

3.4

(a) Left $-\infty$, right $+\infty$.

(b) The asymptote is $x = 3$; the numerator approaches positive 4 while the denominator changes from small negative to small positive.