1.11 Defining Continuity at a Point

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS continuity • continuous • limit • discontinuous

Objective

Build the skills to answer exam questions on **defining continuity at a point**.

You must be able to:

- state the three conditions for **continuity** at a point
- test continuity using $\lim_{x\to a} f(x) = f(a)$

1 Key knowledge and worked examples

Study these first. Each one shows the method for a question type used later.

■ Continuity at a point

f is continuous at a when all three hold:

1. $f(a)$ is defined;
2. $\lim_{x\to a} f(x)$ exists;
3. they are equal: $\lim_{x\to a} f(x) = f(a)$.

■ Example

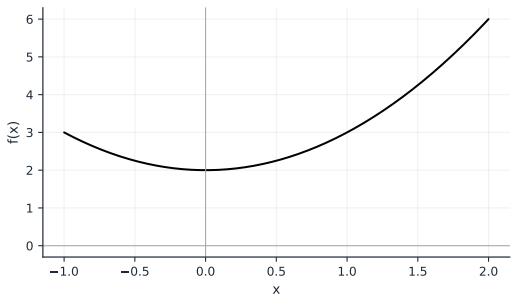
If $\lim_{x\to 2} f(x) = 5$ and $f(2) = 5$, then f is continuous at 2.

■ Key knowledge: Test all continuity conditions

Continuity at a requires a defined point value, an existing two-sided limit and equality between them. Finding only one of these is incomplete.

■ Worked example: Test all continuity conditions

Let $f(x) = x^2 + 2$ for $x \neq 1$ and $f(1) = k$. The limit at 1 is $1^2 + 2 = 3$. Continuity therefore requires $k = 3$. With that assignment the value exists, the two-sided limit exists, and the two are equal.



2 Practice

2.1 State the three conditions for continuity at a. [1]

2.2 State the single equation summarizing continuity at a. [1]

2.3 If $\lim_{x\to 2} f(x) = 5$ and $f(2) = 5$, is f continuous at 2? [2]

3 Exam-style questions

3.1 f is continuous at a if and only if [1]

☐ **A** $f(a)$ is defined
B $\lim_{x\to a} f(x) = f(a)$
C the limit exists only
D f is linear

3.2 If $\lim_{x\to a} f(x)$ exists but $\neq f(a)$, then f is [1]

☐ **A** continuous at a
B discontinuous at a
C linear
D undefined everywhere

3.3 $f(2) = 5$ and $\lim_{x \to 2} f(x) = 5$.

(a) Is $f(2)$ defined? [1]

(b) Does the limit exist? [1]

(c) Is f continuous at 2? [1]

3.4 Let $g(x) = x^2 - x + 1$ for $x \neq 2$ and $g(2) = k$. (a) Find the limit at 2 and the value of k required for continuity. [2]

(b) State the three continuity conditions and explain why your assignment meets them. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $f(a)$ is defined; the limit exists; the limit equals $f(a)$

2.2

- $\lim_{x \to a} f(x) = f(a)$

2.3

- yes – the limit exists, $f(2)$ is defined, and they are equal

3.1 B

- $\lim_{x \to a} f(x) = f(a)$.

3.2 B

- discontinuous at a.

3.3

(a)

- yes

(b)

- yes

(c)

- yes

3.4

(a) Limit $= 4 - 2 + 1 = 3$; $k = 3$.

(b) The point value and limit both exist and are equal at 3.