

Exercise walk-through

9.7

Defining Polar Coordinates and Differentiating in Polar Form

AP Calculus BC

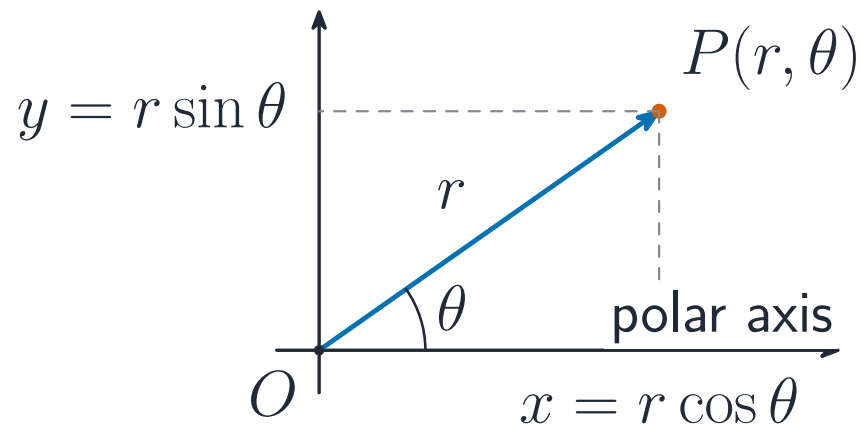
Polar coordinates

Polar coordinates locate a point by its distance r and angle θ :

$$x = r \cos \theta, \quad y = r \sin \theta.$$

A polar curve $r = f(\theta)$ is a **parametric curve in θ** , so

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}.$$

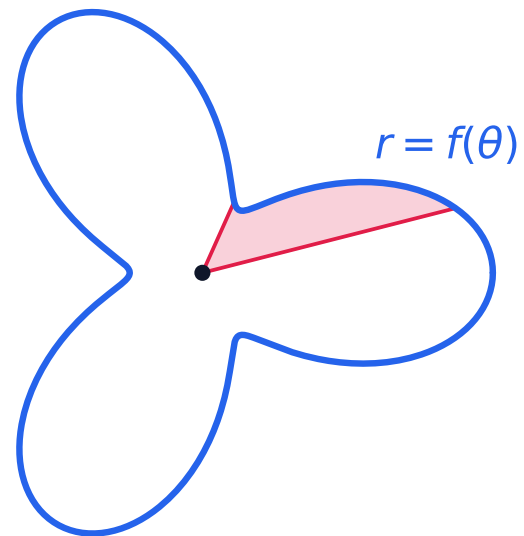


Area of a polar region

The region is a **fan** of thin triangular sectors, each of area $\frac{1}{2}r^2 d\theta$:

$$A = \frac{1}{2} \int_{\alpha}^{\beta} (f(\theta))^2 d\theta.$$

Choosing the right θ -limits – where the curve **starts and finishes tracing** the region – is the main challenge.



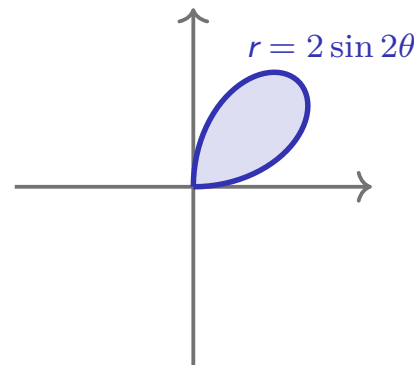
$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$$

Worked example – one petal of a rose

Area of one petal of $r = 2 \sin(2\theta)$, traced for $0 \leq \theta \leq \frac{\pi}{2}$

Using $\sin^2 u = \frac{1}{2}(1 - \cos 2u)$:

$$\begin{aligned} A &= \frac{1}{2} \int_0^{\pi/2} (2 \sin 2\theta)^2 d\theta = \int_0^{\pi/2} (1 - \cos 4\theta) d\theta \\ &= \left[\theta - \frac{\sin 4\theta}{4} \right]_0^{\pi/2} = \frac{\pi}{2}. \end{aligned}$$

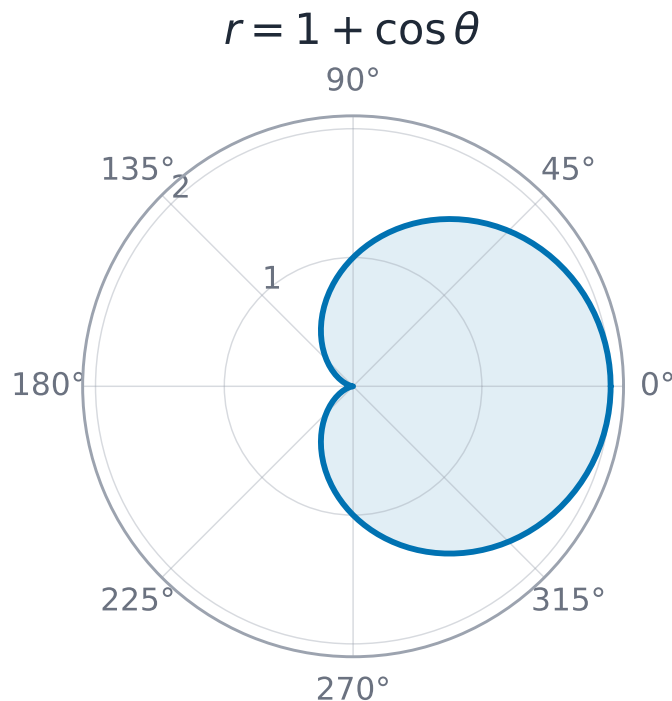


Between two polar curves

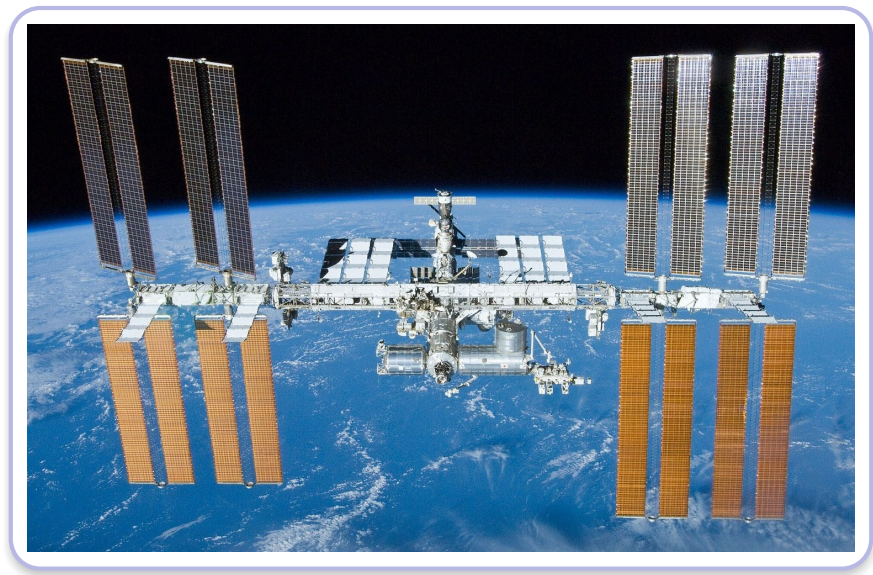
For the region between an outer $r_1(\theta)$ and an inner $r_2(\theta)$, subtract the fans:

$$A = \frac{1}{2} \int_{\alpha}^{\beta} (r_1^2 - r_2^2) d\theta.$$

Subtract the **squares**, not the radii – and find where the curves **intersect** to set α and β .



Defining and Differentiating Parametric Equations



The ISS in orbit: parametric equations describe position as a function of time

Exercises 2.1-2.3

2.1 Convert the polar point $(3, \frac{\pi}{2})$ to Cartesian coordinates. [2]

2.2 Write x and y for the polar curve $r = 2 \cos \theta$. [2]

2.3 Convert the Cartesian point $(0, 5)$ to polar coordinates. [2]

Exercises 2.1-2.3

2.1 Convert the polar point $(3, \frac{\pi}{2})$ to Cartesian coordinates. [2]

2.1

- $x = 3 \cos \frac{\pi}{2} = 0, y = 3 \sin \frac{\pi}{2} = 3: (0, 3)$

2.2 Write x and y for the polar curve $r = 2 \cos \theta$. [2]

2.2

- $x = 2 \cos \theta \cos \theta = 2 \cos^2 \theta, y = 2 \cos \theta \sin \theta$

2.3 Convert the Cartesian point $(0, 5)$ to polar coordinates. [2]

2.3

- $r = 5, \theta = \frac{\pi}{2}: (5, \frac{\pi}{2})$

Exercise 3.1

3.1 For a polar curve $r = f(\theta)$, $x =$

[1]

☐

A $r \sin \theta$

B $r \cos \theta$

C $\frac{r}{\cos \theta}$

D $r + \cos \theta$

Exercise 3.1

3.1 For a polar curve $r = f(\theta)$, $x =$

[1]

☐

A $r \sin \theta$

B $r \cos \theta$

C $\frac{r}{\cos \theta}$

D $r + \cos \theta$

3.1 B

- $x = r \cos \theta$

Exercise 3.2 (a)

3.2 For the polar curve $r = 1 + \cos \theta$:

(a) Write x and y as functions of θ .

[2]

Exercise 3.2 (a)

3.2 For the polar curve $r = 1 + \cos \theta$:

(a) Write x and y as functions of θ .

[2]

(a)

- $x = (1 + \cos \theta) \cos \theta$, $y = (1 + \cos \theta) \sin \theta$

Exercise 3.2 (b)

3.2 For the polar curve $r = 1 + \cos \theta$:

(b) Find the point (in Cartesian coordinates) at $\theta = 0$.

[2]

Exercise 3.2 (b)

3.2 For the polar curve $r = 1 + \cos \theta$:

(b) Find the point (in Cartesian coordinates) at $\theta = 0$.

[2]

(b)

- at $\theta = 0$: $r = 2$, $x = 2$, $y = 0$: $(2, 0)$

Exercise 3.3

3.3 For $r = 2 \sin \theta$, find $\frac{dx}{d\theta}$.

[3]

Exercise 3.3

3.3 For $r = 2 \sin \theta$, find $\frac{dx}{d\theta}$.

[3]

3.3

- $x = 2 \sin \theta \cos \theta = \sin(2\theta)$; $\frac{dx}{d\theta} = 2 \cos(2\theta)$. (Or via product rule on $2 \sin \theta \cos \theta$.)