

Exercise walk-through

9.6

Solving Motion Problems Using Parametric and Vector-Valued Functions

AP Calculus BC

Differentiating a parametric curve

Parametric equations give x and y each as functions of a **parameter** t – so the curve **need not be a function of x** .

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad (dx/dt \neq 0).$$

It is the chain rule: the dt cancels.

$x = t^2, y = t^3 - t$: **slope at $t = 2$**

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2 - 1:$$

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t} \xrightarrow{t=2} \frac{11}{4}.$$

The second derivative – a classic trap

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}.$$

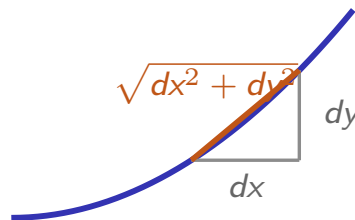
Differentiate the **first derivative** with respect to t , then divide by dx/dt **again**. Use it to test **concavity** of a parametric curve.

The ISS in orbit: parametric equations describe position as a function of time

Arc length of a parametric curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

This is Pythagoras: sum tiny hypotenuses $\sqrt{dx^2 + dy^2}$ over the parameter.



Exercises 2.1·2.3

2.1 For $\vec{v}(t) = \langle 6, 8 \rangle$, find the speed. [1]

2.2 A particle has $\vec{v}(t) = \langle 2t, 2 \rangle$. Set up the total distance travelled over $0 \leq t \leq 3$. [2]

2.3 A particle starts at $\vec{r}(0) = \langle 1, 2 \rangle$ with $\vec{v}(t) = \langle 4, 6t \rangle$. Find $\vec{r}(t)$. [3]

Exercises 2.1·2.3

2.1 For $\vec{v}(t) = \langle 6, 8 \rangle$, find the speed. [1]

2.1

- $\sqrt{36 + 64} = 10$

2.2 A particle has $\vec{v}(t) = \langle 2t, 2 \rangle$. Set up the total distance travelled over $0 \leq t \leq 3$. [2]

2.2

- $\int_0^3 \sqrt{(2t)^2 + 2^2} dt = \int_0^3 \sqrt{4t^2 + 4} dt$

2.3 A particle starts at $\vec{r}(0) = \langle 1, 2 \rangle$ with $\vec{v}(t) = \langle 4, 6t \rangle$. Find $\vec{r}(t)$. [3]

2.3

- $\vec{r}(t) = \langle 4t + 1, 3t^2 + 2 \rangle$

Exercise 3.1

3.1 The total distance a particle travels over $[a, b]$ is

[1]



A $\int_a^b \vec{v} \, dt$

B $|\vec{r}(b) - \vec{r}(a)|$

C $\int_a^b \sqrt{x'^2 + y'^2} \, dt$

D $\vec{r}(b) - \vec{r}(a)$

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3.1 C

- total distance is the arc-length integral of speed

Exercise 3.2 (a)

3.2 A particle moves with $\vec{v}(t) = \langle 3t^2, 2t \rangle$, starting at $\vec{r}(0) = \langle 0, 1 \rangle$.

(a) Find the position $\vec{r}(t)$.

[3]

Exercise 3.2 (a)

3.2 A particle moves with $\vec{v}(t) = \langle 3t^2, 2t \rangle$, starting at $\vec{r}(0) = \langle 0, 1 \rangle$.

(a) Find the position $\vec{r}(t)$.

[3]

(a)

- $\vec{r}(t) = \langle t^3, t^2 + 1 \rangle$

Exercise 3.2 (b)

3.2 A particle moves with $\vec{v}(t) = \langle 3t^2, 2t \rangle$, starting at $\vec{r}(0) = \langle 0, 1 \rangle$.

(b) Find the speed at $t = 1$.

[2]

Exercise 3.2 (b)

3.2 A particle moves with $\vec{v}(t) = \langle 3t^2, 2t \rangle$, starting at $\vec{r}(0) = \langle 0, 1 \rangle$.

(b) Find the speed at $t = 1$.

[2]

(b)

- $\vec{v}(1) = \langle 3, 2 \rangle$; speed $= \sqrt{9 + 4} = \sqrt{13}$

Exercise 3.3

3.3 A particle has $\vec{v}(t) = \langle \cos t, \sin t \rangle$. Set up and evaluate the total distance travelled over $0 \leq t \leq \pi$. [3]

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3.3

- speed = $\sqrt{\cos^2 t + \sin^2 t} = 1$; distance = $\int_0^\pi 1 \, dt = \pi$