

Exercise walk-through

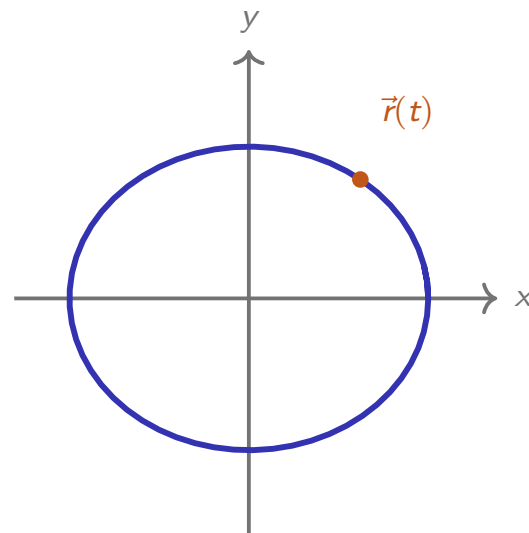
9.5

Integrating Vector-Valued Functions

AP Calculus BC

What we will cover

- 1 Differentiating parametric curves
- 2 Second derivatives & arc length
- 3 Vector-valued functions
- 4 Plane motion
- 5 Polar coordinates
- 6 Area of a polar region



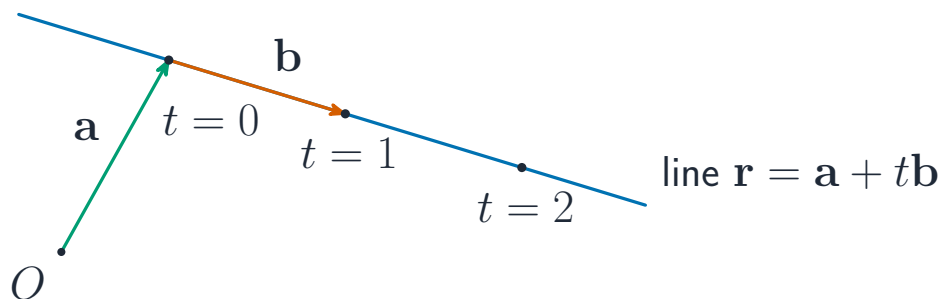
Vector-valued functions

A vector-valued function

$\vec{r}(t) = \langle x(t), y(t) \rangle$ gives a position vector for each t . Differentiate component-wise:

$$\vec{v} = \langle x', y' \rangle, \quad \vec{a} = \langle x'', y'' \rangle.$$

Speed is the magnitude $|\vec{v}| = \sqrt{x'^2 + y'^2}$ – a scalar.



Exam tips

- For **parametric** curves, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; speed is $\sqrt{(dx/dt)^2 + (dy/dt)^2}$.
- A **vector-valued** function carries the same information – differentiate/integrate it component by component.
- In **polar**, convert with $x = r \cos \theta$, $y = r \sin \theta$; area swept is $\frac{1}{2} \int r^2 d\theta$.
- Watch the direction of tracing (the sign of dx/dt) and set correct θ -limits for polar area.
- Eliminate the parameter to recover the ordinary y -vs- x shape when it helps.

Exercises 2.1·2.3

2.1 Find $\int \langle 2t, 3 \rangle dt$ (include the constant vector). [2]

2.2 Given $\vec{a}(t) = \langle 0, -10 \rangle$ and $\vec{v}(0) = \langle 5, 0 \rangle$, find $\vec{v}(t)$. [3]

2.3 Evaluate $\int_0^2 \langle 1, 2t \rangle dt$. [2]

Exercises 2.1·2.3

2.1 Find $\int \langle 2t, 3 \rangle dt$ (include the constant vector). [2]

2.1

- $\langle t^2, 3t \rangle + \vec{C}$

2.2 Given $\vec{a}(t) = \langle 0, -10 \rangle$ and $\vec{v}(0) = \langle 5, 0 \rangle$, find $\vec{v}(t)$. [3]

2.2

- $\vec{v}(t) = \langle C_1, -10t + C_2 \rangle; \vec{v}(0) = \langle 5, 0 \rangle \Rightarrow C_1 = 5, C_2 = 0; \vec{v}(t) = \langle 5, -10t \rangle$

2.3 Evaluate $\int_0^2 \langle 1, 2t \rangle dt$. [2]

2.3

- $\langle [t]_0^2, [t^2]_0^2 \rangle = \langle 2, 4 \rangle$

Exercise 3.1

3.1 To integrate a vector-valued function you

[1]

☐

A integrate each component separately

B take the magnitude first

C differentiate it

D add the components then integrate

Exercise 3.1

3.1 To integrate a vector-valued function you

[1]

☐

A integrate each component separately

B take the magnitude first

C differentiate it

D add the components then integrate

3.1 A

- integrate each component separately

Exercise 3.2 (a)

3.2 A particle has acceleration $\vec{a}(t) = \langle 6t, 2 \rangle$, with $\vec{v}(0) = \langle 0, 1 \rangle$.

(a) Find $\vec{v}(t)$.

[3]

Exercise 3.2 (a)

3.2 A particle has acceleration $\vec{a}(t) = \langle 6t, 2 \rangle$, with $\vec{v}(0) = \langle 0, 1 \rangle$.

(a) Find $\vec{v}(t)$.

[3]

(a)

- $\vec{v}(t) = \langle 3t^2 + C_1, 2t + C_2 \rangle$; $\vec{v}(0) = \langle 0, 1 \rangle \Rightarrow C_1 = 0, C_2 = 1$; $\vec{v}(t) = \langle 3t^2, 2t + 1 \rangle$

Exercise 3.2 (b)

3.2 A particle has acceleration $\vec{a}(t) = \langle 6t, 2 \rangle$, with $\vec{v}(0) = \langle 0, 1 \rangle$.

(b) Find $\vec{v}(1)$.

[1]

Exercise 3.2 (b)

3.2 A particle has acceleration $\vec{a}(t) = \langle 6t, 2 \rangle$, with $\vec{v}(0) = \langle 0, 1 \rangle$.

(b) Find $\vec{v}(1)$.

[1]

(b)

- $\vec{v}(1) = \langle 3, 3 \rangle$

Exercise 3.3

3.3 A particle has velocity $\vec{v}(t) = \langle 2t, \cos t \rangle$ and starts at $\vec{r}(0) = \langle 1, 0 \rangle$. Find the position $\vec{r}(t)$. [4]

Exercise 3.3

3.3 A particle has velocity $\vec{v}(t) = \langle 2t, \cos t \rangle$ and starts at $\vec{r}(0) = \langle 1, 0 \rangle$. Find the position $\vec{r}(t)$. [4]

3.3

- $\vec{r}(t) = \langle t^2 + C_1, \sin t + C_2 \rangle$; $\vec{r}(0) = \langle 1, 0 \rangle \Rightarrow C_1 = 1, C_2 = 0$; $\vec{r}(t) = \langle t^2 + 1, \sin t \rangle$