

Exercise walk-through

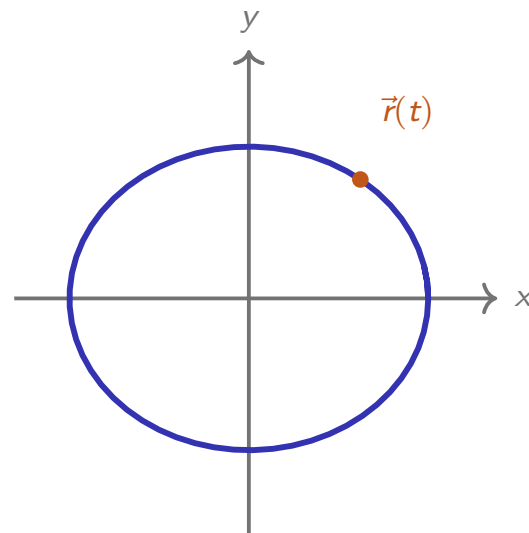
9.4

Defining and Differentiating Vector-Valued Functions

AP Calculus BC

What we will cover

- 1 Differentiating parametric curves
- 2 Second derivatives & arc length
- 3 Vector-valued functions
- 4 Plane motion
- 5 Polar coordinates
- 6 Area of a polar region



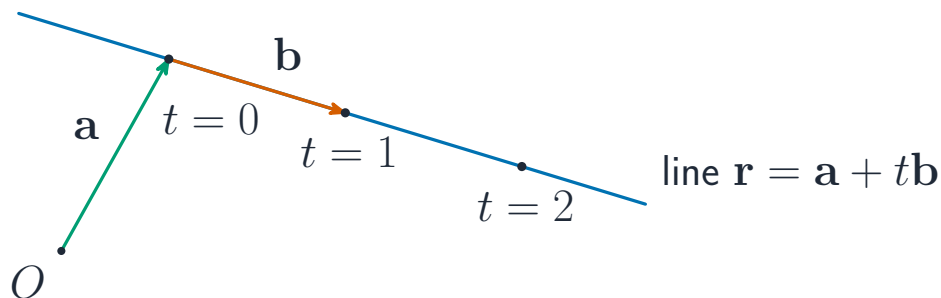
Vector-valued functions

A vector-valued function

$\vec{r}(t) = \langle x(t), y(t) \rangle$ gives a position vector for each t . Differentiate component-wise:

$$\vec{v} = \langle x', y' \rangle, \quad \vec{a} = \langle x'', y'' \rangle.$$

Speed is the magnitude $|\vec{v}| = \sqrt{x'^2 + y'^2}$ – a scalar.



Exam tips

- For **parametric** curves, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$; speed is $\sqrt{(dx/dt)^2 + (dy/dt)^2}$.
- A **vector-valued** function carries the same information – differentiate/integrate it component by component.
- In **polar**, convert with $x = r \cos \theta$, $y = r \sin \theta$; area swept is $\frac{1}{2} \int r^2 d\theta$.
- Watch the direction of tracing (the sign of dx/dt) and set correct θ -limits for polar area.
- Eliminate the parameter to recover the ordinary y -vs- x shape when it helps.

Exercises 2.1·2.3

2.1 For $\vec{r}(t) = \langle t^3, t^2 \rangle$, find $\vec{v}(t)$. [2]

2.2 For $\vec{v}(t) = \langle 2t, 3 \rangle$, find the speed at $t = 2$. [2]

2.3 For $\vec{r}(t) = \langle \cos t, \sin t \rangle$, find $\vec{a}(t)$. [2]

Exercises 2.1·2.3

2.1 For $\vec{r}(t) = \langle t^3, t^2 \rangle$, find $\vec{v}(t)$. [2]

2.1

- $\vec{v}(t) = \langle 3t^2, 2t \rangle$

2.2 For $\vec{v}(t) = \langle 2t, 3 \rangle$, find the speed at $t = 2$. [2]

2.2

- $\vec{v}(2) = \langle 4, 3 \rangle$; speed $= \sqrt{16 + 9} = 5$

2.3 For $\vec{r}(t) = \langle \cos t, \sin t \rangle$, find $\vec{a}(t)$. [2]

2.3

- $\vec{v} = \langle -\sin t, \cos t \rangle$; $\vec{a} = \langle -\cos t, -\sin t \rangle$

Exercise 3.1

3.1 The speed of a particle with velocity $\vec{v} = \langle x', y' \rangle$ is

[1]

☐

A $x' + y'$

B $\sqrt{x'^2 + y'^2}$

C $\langle x', y' \rangle$

D $x'y'$

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A $x' + y'$

B $\sqrt{x'^2 + y'^2}$

C $\langle x', y' \rangle$

D $x'y'$

3.1 B

- speed is the magnitude $\sqrt{x'^2 + y'^2}$

Exercise 3.2 (a)

3.2 A particle has position $\vec{r}(t) = \langle t^2 - 1, 2t \rangle$.

(a) Find $\vec{v}(t)$ and $\vec{a}(t)$.

[2]

Exercise 3.2 (a)

3.2 A particle has position $\vec{r}(t) = \langle t^2 - 1, 2t \rangle$.

(a) Find $\vec{v}(t)$ and $\vec{a}(t)$.

[2]

(a)

- $\vec{v} = \langle 2t, 2 \rangle, \vec{a} = \langle 2, 0 \rangle$

Exercise 3.2 (b)

3.2 A particle has position $\vec{r}(t) = \langle t^2 - 1, 2t \rangle$.

(b) Find the speed at $t = 1$.

[2]

Exercise 3.2 (b)

3.2 A particle has position $\vec{r}(t) = \langle t^2 - 1, 2t \rangle$.

(b) Find the speed at $t = 1$.

[2]

(b)

- $\vec{v}(1) = \langle 2, 2 \rangle$; speed $= \sqrt{4 + 4} = 2\sqrt{2}$

Exercise 3.3

3.3 For $\vec{r}(t) = \langle e^t, e^{-t} \rangle$, find the velocity and speed at $t = 0$.

[4]

Exercise 3.3

3.3 For $\vec{r}(t) = \langle e^t, e^{-t} \rangle$, find the velocity and speed at $t = 0$.

[4]

3.3

- $\vec{v} = \langle e^t, -e^{-t} \rangle$; at $t = 0$, $\vec{v} = \langle 1, -1 \rangle$; speed $= \sqrt{1 + 1} = \sqrt{2}$