

Exercise walk-through

9.2

Second Derivatives of Parametric Equations

AP Calculus BC

Differentiating a parametric curve

Parametric equations give x and y each as functions of a **parameter** t – so the curve **need not be a function of x** .

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad (dx/dt \neq 0).$$

It is the chain rule: the dt cancels.

$x = t^2, y = t^3 - t$: **slope at $t = 2$**

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2 - 1:$$

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t} \xrightarrow{t=2} \frac{11}{4}.$$

The second derivative – a classic trap

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}.$$

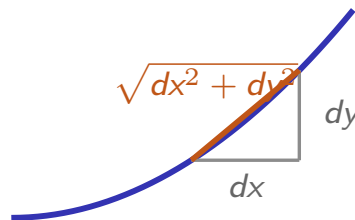
Differentiate the **first derivative** with respect to t , then divide by dx/dt **again**. Use it to test **concavity** of a parametric curve.

The ISS in orbit: parametric equations describe position as a function of time

Arc length of a parametric curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

This is Pythagoras: sum tiny hypotenuses $\sqrt{dx^2 + dy^2}$ over the parameter.



Exercises 2.1-2.2

2.1 State the formula for $\frac{d^2y}{dx^2}$ for a parametric curve. [1]

2.2 For $x = t^2$, $y = t^3$, given $\frac{dy}{dx} = \frac{3t}{2}$, find $\frac{d}{dt} \left(\frac{dy}{dx} \right)$. [1]

Exercises 2.1·2.2

2.1 State the formula for $\frac{d^2y}{dx^2}$ for a parametric curve. [1]

2.1

- $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dx)}{dx/dt}$$

2.2 For $x = t^2$, $y = t^3$, given $\frac{dy}{dx} = \frac{3t}{2}$, find $\frac{d}{dt}\left(\frac{dy}{dx}\right)$. [1]

2.2

- $$\frac{d}{dt}\left(\frac{3t}{2}\right) = \frac{3}{2}$$

Exercises 2.3-3.1

2.3 Complete: find $\frac{d^2y}{dx^2}$ for that curve.

[2]

3.1 The second derivative of a parametric curve is

[1]

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A $\frac{d^2y/dt^2}{d^2x/dt^2}$

B $\frac{\frac{d}{dt}(dy/dx)}{dx/dt}$

C $\frac{dy/dt}{dx/dt}$

D $\frac{d}{dt}\left(\frac{dy}{dx}\right)$

Exercises 2.3-3.1

2.3 Complete: find $\frac{d^2y}{dx^2}$ for that curve.

[2]

2.3

- $\frac{d^2y}{dx^2} = \frac{3/2}{2t} = \frac{3}{4t}$

3.1 The second derivative of a parametric curve is

[1]



A $\frac{d^2y/dt^2}{d^2x/dt^2}$

B $\frac{\frac{d}{dt}(dy/dx)}{dx/dt}$

C $\frac{dy/dt}{dx/dt}$

D $\frac{d}{dt}\left(\frac{dy}{dx}\right)$

3.1 B

- differentiate dy/dx w.r.t. t , then divide by dx/dt

Exercise 3.2 (a)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(a) Find $\frac{dy}{dx}$.

[1]

Exercise 3.2 (a)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(a) Find $\frac{dy}{dx}$.

[1]

(a)

$$\bullet \frac{dy}{dx} = \frac{3t^2 - 3}{1} = 3t^2 - 3$$

Exercise 3.2 (b)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(b) Find $\frac{d^2y}{dx^2}$.

[2]

Exercise 3.2 (b)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(b) Find $\frac{d^2y}{dx^2}$.

[2]

(b)

- $\frac{d}{dt}(3t^2 - 3) = 6t$; divide by $dx/dt = 1$: $\frac{d^2y}{dx^2} = 6t$

Exercise 3.2 (c)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(c) State the concavity at $t = 1$.

[1]

Exercise 3.2 (c)

3.2 A curve has $x = t$, $y = t^3 - 3t$.

(c) State the concavity at $t = 1$.

[1]

(c)

- at $t = 1$, $6 > 0$: concave up

Exercise 3.3

3.3 For $x = t^2$, $y = t^2 + t$, find $\frac{d^2y}{dx^2}$ at $t = 1$.

[4]

Exercise 3.3

3.3 For $x = t^2$, $y = t^2 + t$, find $\frac{d^2y}{dx^2}$ at $t = 1$. [4]

3.3

- $\frac{dy}{dx} = \frac{2t+1}{2t}$; $\frac{d}{dt}\left(\frac{2t+1}{2t}\right) = \frac{d}{dt}\left(1 + \frac{1}{2t}\right) = -\frac{1}{2t^2}$; divide by $2t$: $\frac{d^2y}{dx^2} = -\frac{1}{4t^3}$; at $t = 1$:
 $-\frac{1}{4}$