

Exercise walk-through

9.1

Defining and Differentiating Parametric Equations

AP Calculus BC

Differentiating a parametric curve

Parametric equations give x and y each as functions of a **parameter** t – so the curve **need not be a function of x** .

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad (dx/dt \neq 0).$$

It is the chain rule: the dt cancels.

$x = t^2, y = t^3 - t$: **slope at $t = 2$**

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2 - 1:$$

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t} \xrightarrow{t=2} \frac{11}{4}.$$

The second derivative – a classic trap

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}.$$

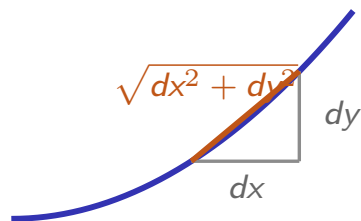
Differentiate the **first derivative** with respect to t , then divide by dx/dt **again**. Use it to test **concavity** of a parametric curve.

The ISS in orbit: parametric equations describe position as a function of time

Arc length of a parametric curve

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

This is Pythagoras: sum tiny hypotenuses $\sqrt{dx^2 + dy^2}$ over the parameter.



Exercises 2.1·2.3

2.1 For $x = t^2$, $y = t^3$, state $\frac{dx}{dt}$ and $\frac{dy}{dt}$. [2]

2.2 For $x = \cos t$, $y = \sin t$, find $\frac{dy}{dx}$. [2]

2.3 For $x = t^2$, $y = 4t$, find the slope at $t = 1$. [2]

Exercises 2.1-2.3

2.1 For $x = t^2$, $y = t^3$, state $\frac{dx}{dt}$ and $\frac{dy}{dt}$. [2]

2.1

- $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2$

2.2 For $x = \cos t$, $y = \sin t$, find $\frac{dy}{dx}$. [2]

2.2

- $\frac{dy}{dx} = \frac{\cos t}{-\sin t} = -\cot t$

2.3 For $x = t^2$, $y = 4t$, find the slope at $t = 1$. [2]

2.3

- $\frac{dy}{dx} = \frac{4}{2t} = \frac{2}{t}$; at $t = 1$ slope = 2

Exercise 3.1

3.1 For a parametric curve, $\frac{dy}{dx} =$

[1]

☐

A $\frac{dx/dt}{dy/dt}$

B $\frac{dy/dt}{dx/dt}$

C $\frac{dy}{dt} \cdot \frac{dx}{dt}$

D $\frac{d^2y}{dt^2}$

Exercise 3.1

3.1 For a parametric curve, $\frac{dy}{dx} =$

[1]

- ☐ A $\frac{dx/dt}{dy/dt}$
- ☐ B $\frac{dy/dt}{dx/dt}$
- ☐ C $\frac{dy}{dt} \cdot \frac{dx}{dt}$
- ☐ D $\frac{d^2y}{dt^2}$

3.1 B

• $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

Exercise 3.2 (a)

3.2 A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(a) Find $\frac{dy}{dx}$ in terms of t .

[2]

Exercise 3.2 (a)

3.2 A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(a) Find $\frac{dy}{dx}$ in terms of t .

[2]

(a)

- $\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$

Exercise 3.2 (b)

3.2 A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(b) Find the value(s) of t where the tangent is horizontal.

[2]

Exercise 3.2 (b)

3.2 A curve is given by $x = t^2 - 1$, $y = t^3 - 3t$.

(b) Find the value(s) of t where the tangent is horizontal.

[2]

(b)

- horizontal where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$

Exercise 3.3

3.3 For $x = e^t$, $y = e^{2t}$, find the slope of the tangent line at $t = 0$.

[3]

Exercise 3.3

3.3 For $x = e^t$, $y = e^{2t}$, find the slope of the tangent line at $t = 0$.

[3]

3.3

- $\frac{dx}{dt} = e^t$, $\frac{dy}{dt} = 2e^{2t}$; $\frac{dy}{dx} = \frac{2e^{2t}}{e^t} = 2e^t$; at $t = 0$, slope = 2