

Exercise walk-through

8.8

Volumes with Cross Sections: Triangles and Semicircles

AP Calculus BC

You must be able to

- use $A = \frac{\sqrt{3}}{4}s^2$ for an **equilateral triangle** slice
- use $A = \frac{\pi}{8}s^2$ for a **semicircular** slice (diameter = s)
- substitute the area formula and integrate

1

The method

Method — Equilateral-triangle cross sections

A slice that is an equilateral triangle with side s has area $A = \frac{\sqrt{3}}{4}s^2$. If $s = f(x) - g(x)$, then

$$V = \int_a^b \frac{\sqrt{3}}{4} (f(x) - g(x))^2 dx.$$

Exercises 2.1-2.3

2.1 State the area of an equilateral triangle with side s . [1]

2.2 State the area of a semicircle with diameter s . [1]

2.3 A solid has equilateral-triangle cross sections with side $s = x$ on $[0, 2]$. Find its volume. [3]

Exercises 2.1·2.3

2.1 State the area of an equilateral triangle with side s .

[1]

2.1

- $A = \frac{\sqrt{3}}{4}s^2$

2.2 State the area of a semicircle with diameter s .

[1]

2.2

- $A = \frac{\pi}{8}s^2$

2.3 A solid has equilateral-triangle cross sections with side $s = x$ on $[0, 2]$. Find its volume.

[3]

2.3

- $V = \int_0^2 \frac{\sqrt{3}}{4}x^2 dx = \frac{\sqrt{3}}{4} \left[\frac{x^3}{3} \right]_0^2 = \frac{\sqrt{3}}{4} \cdot \frac{8}{3} = \frac{2\sqrt{3}}{3}$

Exercise 3.1

3.1 A semicircular cross section with diameter s has area

[1]



A $\frac{\pi}{2}s^2$

B $\frac{\pi}{4}s^2$

C $\frac{\pi}{8}s^2$

D πs^2

Exercise 3.1

3.1 A semicircular cross section with diameter s has area

[1]



A $\frac{\pi}{2}s^2$

B $\frac{\pi}{4}s^2$

C $\frac{\pi}{8}s^2$

D πs^2

3.1 C

- radius $s/2$ gives $\frac{1}{2}\pi(s/2)^2 = \frac{\pi}{8}s^2$

Exercise 3.2 (a)

3.2 The base of a solid is bounded by $y = 2 - x$ and the axes in the first quadrant. Cross sections perpendicular to the x -axis are equilateral triangles.

(a) Write the side length s as a function of x .

[1]

Exercise 3.2 (a)

3.2 The base of a solid is bounded by $y = 2 - x$ and the axes in the first quadrant. Cross sections perpendicular to the x -axis are equilateral triangles.

(a) Write the side length s as a function of x . [1]

(a)

- $s = 2 - x$

Exercise 3.2 (b)

3.2 The base of a solid is bounded by $y = 2 - x$ and the axes in the first quadrant. Cross sections perpendicular to the x -axis are equilateral triangles.

(b) Set up the volume integral.

[2]

Exercise 3.2 (b)

3.2 The base of a solid is bounded by $y = 2 - x$ and the axes in the first quadrant. Cross sections perpendicular to the x -axis are equilateral triangles.

(b) Set up the volume integral.

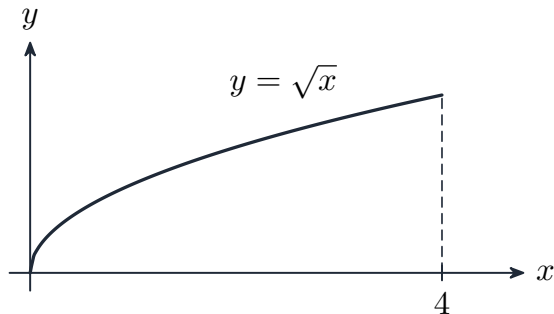
[2]

(b)

- $\int_0^2 \frac{\sqrt{3}}{4} (2 - x)^2 dx$

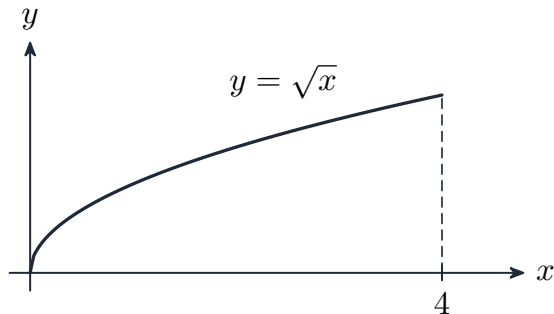
Exercise 3.3

3.3 The base is the region between $y = \sqrt{x}$ and the x -axis on $[0, 4]$. Cross sections perpendicular to the x -axis are semicircles with diameter across the region. Set up and evaluate the volume. [4]



Exercise 3.3

3.3 The base is the region between $y = \sqrt{x}$ and the x -axis on $[0, 4]$. Cross sections perpendicular to the x -axis are semicircles with diameter across the region. Set up and evaluate the volume. [4]



3.3

- $s = \sqrt{x}; V = \int_0^4 \frac{\pi}{8} (\sqrt{x})^2 dx = \frac{\pi}{8} \int_0^4 x dx = \frac{\pi}{8} \left[\frac{x^2}{2} \right]_0^4 = \frac{\pi}{8} \cdot 8 = \pi$